HOMEWORK 4 - MATH 160
DUE DATE: Tuesday, September 22
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Two out of the ten problems will be chosen at random and graded for a total of 20 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Consider the graph of $y = f(x)$ of Exercise 2 on page 126 of your textbook.
   (a) Find $\lim_{x\to 3^-} f(x), \lim_{x\to 3^+} f(x)$.
   (b) Find $\lim_{x\to 3} f(x)$ and $f(3)$.
   (c) Is $f(x)$ continuous at $x = 3$? Explain.
2. Consider the graph of $y = f(x)$ of Exercise 8 on page 126 of your textbook.
   (a) Find $\lim_{x\to 0^-} f(x), \lim_{x\to 0^+} f(x)$.
   (b) Find $\lim_{x\to 0} f(x)$.
   (c) Is $f(x)$ continuous at $x = 0$? Explain.
3. Suppose $f(x) = \begin{cases} 2x-4, & \text{if } x < 1 \\ 3, & \text{if } x = 1 \\ \frac{x-5}{x+1}, & \text{if } x > 1 \end{cases}$.
   (a) Find $\lim_{x\to 1} f(x)$.
   (b) Is $f(x)$ continuous at $x = 1$? Explain.
4. Find $f'(a)$ if $f(x) = -x^2 + 3x$.
5. Find an equation for the tangent line to the graph of $f(x) = \sqrt{x-3}$ at $x = 4$.
6. Find the point(s) $x = a$ at which the tangent line to the graph of $f(x) = \frac{3}{2x}$ has slope $m = -\frac{2}{3}$.
7. A hot air balloon rises vertically from the ground so that its height after t sec is $h = \frac{1}{2}t^2 + \frac{1}{2}t$ ft, $0 \leq t \leq 60$.
   (a) What is the average velocity of the balloon between $t = 0$ and $t = 40$?
   (b) What is the instantaneous velocity of the balloon at $t = 40$?

   In the previous problems you ought to evaluate the derivatives using the limit definition. In the following two problems, you are supposed to use the rules of differentiation.
8. Find $f'(x)$ using the rules of differentiation, if
   (a) $f(x) = \frac{5}{4}x^{4/5}$
   (b) $f(x) = x^4 - 2x^3 + 7x$
   (c) $f(x) = \frac{5}{x^3} - \frac{2}{x^2} - \frac{1}{x} + 200$
   (d) $f(x) = \frac{3}{x^3} + \frac{4}{\sqrt{x}} + 1$
9. Find the point(s) on the graph of $f(x) = x^3 + 1$, where the slope of the tangent line is equal to 12. Then find the equation(s) of the tangent line(s).
10. No tenth problem this week!