

# HOMEWORK 6 - MATH 160

DUE DATE: Tuesday, October 13

INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Two out of the ten problems will be chosen at random and graded for a total of 20 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Use implicit differentiation to find  $y'$  if

(a)  $x^2y^3 - 2xy^2 = 5$

(b)  $\frac{x-y}{2x+3y} = 2x$

2. At a distance of 50 feet from the pad, a man observes a helicopter taking off from a heliport. If the helicopter lifts off vertically and is rising at a speed of 44 feet per second when it is at an altitude of 120 feet, how fast is the distance between the helicopter and the man changing at that instant?
3. George is blowing air into a soap bubble at the rate of 8 cubic centimeters per second. Assuming that the bubble is spherical, how fast is its radius changing at the instant of time when the radius is 10 centimeters? How fast is the surface area of the bubble changing at that instant of time?

**Hint:** The volume of a sphere is  $V = \frac{4}{3}\pi r^3$  and the surface area is  $S = 4\pi r^2$ .

4. Study the function  $f(x) = x^4 - 4x^3 + 10$  with respect to monotonicity, i.e., find the intervals over which it is increasing and over which it is decreasing.
5. Study the function  $f(x) = \frac{2x}{x^2+1}$  with respect to monotonicity, i.e., find the intervals over which it is increasing and over which it is decreasing.
6. Find the relative maxima and the relative minima of the function  $f(x) = x^2 + 3x + 8$ .
7. Find the relative maxima and the relative minima of  $f(x) = 3x^4 - 2x^3 + 4$ .
8. Find the relative maxima and the relative minima of  $f(x) = \frac{x}{x^2-1}$ .
9. Find the domain of  $f(x) = -\sqrt{4-x^2}$ . Show that  $f(x)$  is concave upward over its entire domain.

**Hint:** To find the domain, please consult how to use the sign table to solve inequalities. We did this during the first couple of weeks in class.

10. Study the following functions with respect to concavity, i.e., find the intervals over which they are concave up and over which they are concave down:

(a)  $f(x) = 3x^4 - 6x^3 + x - 8$

(b)  $f(x) = x + \frac{1}{x^2}$