PRACTICE FINAL EXAM - MATH 160

DATE: Tuesday, December 15

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 25 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

NAME:_____

Section:_____

	Score
Problem 1	
Problem 2	
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Problem 6	
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Problem 8	
TOTAL	

1. (a) Find an equation for the line that passes through $(-3, -2)$ and is parallel to the line passing through $(-2, -4)$ and $(1, 5)$. (5 points)

   (b) Sketch (nicely) the graph of the function $f(x) = \begin{cases} x + 2, & \text{if } x \leq -1 \\ -x^2 + 1, & \text{if } x > -1 \end{cases}$ (10 points)

   (c) Find the following limits, if they exist:

      i. $\lim_{x \to -5} \frac{x^2 - 25}{x + 5}$ (5 points)

      ii. $\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2}$ (5 points)

2. (a) Suppose $f(x) = \begin{cases} 2x-4, & \text{if } x < 1 \\ 3, & \text{if } x = 1 \\ \frac{x-5}{x+1}, & \text{if } x > 1 \end{cases}$.

   i. Find $\lim_{x \to 1} f(x)$. (10 points)

   ii. Is $f(x)$ continuous at $x = 1$? Explain. (5 points)

   (b) Use the limit definition of the derivative to find the slope of the tangent line to graph of $f(x) = \sqrt{x-3}$ at $x = 4$. Then find an equation for the tangent line. (10 points)

3. (a) Find $f'(x)$ using the rules of differentiation:

   i. $f(x) = \frac{3}{x^3} + \frac{4}{\sqrt{x}} + 1$ (5 points)

   ii. $f(x) = (x^3 + 2x + 1)(2 + \frac{1}{x^2})$ (5 points)

   iii. $f(x) = \frac{x+1}{2x^2+2x+3}$ (5 points)

   iv. $f(x) = \frac{4}{\sqrt[3]{2x^2+x}}$ (5 points)

   (b) Use implicit differentiation to find y' if $x^2y^3 - 2xy^2 = 5$. (5 points)

4. (a) George is blowing air into a soap bubble at the rate of 8 cubic centimeters per second. Assuming that the bubble is spherical, how fast is its radius changing at the instant of time when the radius is 10 centimeters? How fast is the surface area of the bubble changing at that instant of time? (5 points)

   Hint: The volume of a sphere is $V = \frac{4}{3}\pi r^3$ and the surface area is $S = 4\pi r^2$.

   (b) Find the relative maxima and the relative minima of the function $f(x) = x^2 + 3x + 8$. (5 points)

   (c) Study $f(x) = x + \frac{1}{x^2}$ with respect to concavity, i.e., find the intervals over which it is concave up and over which it is concave down. (5 points)

   (d) A rectangular box is to have a square base and a volume of 20 cubic feet. If the material for the base costs 30 cents per square foot, the material for the sides costs 10 cents per square foot and the material for the top costs 20 cents per square foot, determine the dimensions of the box that can be constructed at minimum cost. (10 points)

5. (a) On the same coordinate axes sketch the graphs of $f(x) = 4^x$ and $g(x) = \log_{\frac{1}{3}} x$. (5 points)

   (b) Write as a single logarithm or expand and simplify, as appropriate: (5 points)

      i. $\ln 2 + \frac{1}{2}\ln(x+1) - 2\ln(1+\sqrt{x})$

      ii. $\log \frac{\sqrt{x+1}}{x^2+1}$

   (c) Solve the following equations for x: (5 points)

      i. $27^x = (\frac{1}{81})^{x-2}$

      ii. $\log_2 x + \log_2(x-6) = 4$

   (d) Find the derivatives of the following functions:

      i. $f(x) = (4 - e^{-5x})^3$ (5 points)

      ii. $f(x) = \ln \frac{x+1}{x-1}$ (5 points)

6. (a) Find the indefinite integrals:
   i. $\int x^{-2}(1 - x^2 + x^4)dx$ (5 points)
   ii. $\int \frac{x^2-1}{x^3-3x+1}dx$ (5 points)

   (b) Find $y = f(x)$ by solving the differential equation: $f'(x) = 1 + e^x + \frac{1}{x}$; $f(1) = 3 + e$. (5 points)

   (c) Find the function $y = f(x)$, given that the slope of the tangent line to the graph of $y = f(x)$ at any point $(x, f(x))$ is $f'(x) = 1 - \frac{2x}{x^2+1}$ and that its graph passes through the point $(0, 2)$. (10 points)

7. (a) Find the indefinite integrals:
   i. $\int_0^1 \frac{e^x}{1+e^x} dx$; (5 points)
   ii. $\int_1^2 \frac{\ln x}{4x} dx$. (5 points)

   (b) Find the area of the region lying under the graph of $f(x) = xe^{x^2}$ between $x = 0$ and $x = 2$. (5 points)

   (c) Find the area of the region trapped between the graphs of the functions $f(x) = x^3$ and $g(x) = x$. (10 points)

8. (a) Compute the following indefinite integrals:
   i. $\int x^2 e^{-x} dx$ (Need to integrate twice) (10 points)
   ii. $\int x \ln(x+1) dx$ (First substitution; then by-parts) (10 points)

   (b) Find the function f given that the slope of the tangent line to the graph of f at any point $(x, f(x))$ is $x\sqrt{x+1}$ and that the graph passes through the point $(3, 6)$. (10 points)