HOMEWORK 6: SOLUTIONS - MATH 110
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Problem 1 *Solve the* 2×2 *system of equations*

$$\left\{ \begin{array}{rcr} 2x - y & = & 3 \\ x - 3y & = & -1 \end{array} \right\}.$$

Solution:

Solve the first equation for y. We get $y = 2x - 3$. Substitute this value for y in the second equation: $x - 3(2x - 3) = -1$, i.e., $x - 6x + 9 = -1$, whence $-5x = -10$. Therefore $x = 2$. Back substituting in our solution for y, we obtain $y = 2 \cdot 2 - 3 = 4 - 3 = 1$. Hence $(2, 1)$ is the solution to the given system. ∎

Problem 2 *Solve the* 2×2 *systems*

$$\left\{ \begin{array}{lcr} x + 4y & = & 2 \\ 3x + 12y & = & 8 \end{array} \right\} \quad \textit{and} \quad \left\{ \begin{array}{lcr} x - 2y & = & 3 \\ 5x - 10y & = & 15 \end{array} \right\}$$

Solution:

We have for the first system $x = -4y + 2$, whence, using this in the second equation, $3(-4y + 2) + 12y = 8$, i.e, $-12y + 6 + 12y = 8$, which yields, $6 = 8$. Thus we have an inconsistent system with no solutions.

For the second system we obtain $x = 2y + 3$, whence, by substituting this value for x in the second equation, $5(2y + 3) - 10y = 15$, i.e., $10y + 15 - 10y = 15$, which, finally, yields $0 = 0$. Therefore one of the two equations is redundant and we obtain infinitely many solutions

$$(2y + 3, y), \quad y \text{ any real number.}$$

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Problem 3 *Use the method of* **Gauss Elimination** *that was presented in class to solve the system of equations*

$$\left\{ \begin{array}{lcr} x - y & = & 3 \\ -4x + y & = & 3 \end{array} \right\}.$$

Solution:

We have

$$\left\{ \begin{array}{lcr} x - y & = & 3 \\ -4x + y & = & 3 \end{array} \right\} \Longrightarrow \left\{ \begin{array}{lcr} x - y & = & 3 \\ -3y & = & 15 \end{array} \right\} \Longrightarrow$$

$$\left\{ \begin{array}{lcr} x - y & = & 3 \\ y & = & -5 \end{array} \right\} \Longrightarrow \left\{ \begin{array}{lcr} x & = & -2 \\ y & = & -5 \end{array} \right\}$$

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Problem 4 *Two college students organizing a party, in which they are expecting 18 participants, went to their neighborhood store and bought a few six packs of* CoorsLight® *and a few six packs of* Budweiser®. *The six pack of Coors costs \$3, whereas the six pack of Budweiser costs \$5. The two friends joined finances and reasoning and they figured that they have available \$35 for the beer and that each of the participants will probably consume around 3 bottles of whichever beer they are offered. Can you help them decide how many six packs of each beer they should buy?*

Solution:

Let c be the 6-packs of Coors and b the 6-packs of Bud that they will buy. Then, price considerations give the equation $3c + 5b = 35$. The number of bottles that will be purchased, which equal $6c + 6b$, must equal the number of bottles that will be consumed, which is $3 \cdot 18 = 54$. Hence, the second equuation is $6c + 6b = 54$. This yields the system

$$\left\{ \begin{array}{rcl} 3c + 5b & = & 35 \\ 6c + 6b & = & 54 \end{array} \right\}$$

We solve this system by Gauss elimination

$$\left\{ \begin{array}{rcl} 3c + 5b & = & 35 \\ 6c + 6b & = & 54 \end{array} \right\} \Longrightarrow \left\{ \begin{array}{rcl} c + \frac{5}{3}b & = & \frac{35}{3} \\ 6c + 6b & = & 54 \end{array} \right\} \Longrightarrow$$

$$\left\{ \begin{array}{lcl} c + \frac{5}{3}b & = & \frac{35}{3} \\ -4b & = & -16 \end{array} \right\} \Longrightarrow \left\{ \begin{array}{lcl} c + \frac{5}{3}b & = & \frac{35}{3} \\ b & = & 4 \end{array} \right\} \Longrightarrow$$

$$\left\{ \begin{array}{rcl} c & = & 5 \\ b & = & 4 \end{array} \right\}$$

Hence, they will have to buy 5 6-packs of Coors and 4 6-packs of Bud. ■

Problem 5 *Let*

$$A = \left[\begin{array}{rr} 1 & -2 \\ 3 & 5 \end{array} \right], \quad B = \left[\begin{array}{rr} -2 & -1 \\ 5 & 7 \end{array} \right].$$

Compute $A + B, A - B$ *and* $2A - 3B$.

Solution:

$$A + B = \left[\begin{array}{rr} 1 & -2 \\ 3 & 5 \end{array} \right] + \left[\begin{array}{rr} -2 & -1 \\ 5 & 7 \end{array} \right] = \left[\begin{array}{cc} 1 + (-2) & -2 + (-1) \\ 3 + 5 & 5 + 7 \end{array} \right] = \left[\begin{array}{rr} -1 & -3 \\ 8 & 12 \end{array} \right].$$

$$A - B = \left[\begin{array}{rr} 1 & -2 \\ 3 & 5 \end{array} \right] - \left[\begin{array}{rr} -2 & -1 \\ 5 & 7 \end{array} \right] = \left[\begin{array}{cc} 1 - (-2) & -2 - (-1) \\ 3 - 5 & 5 - 7 \end{array} \right] = \left[\begin{array}{rr} 3 & 1 \\ -2 & -2 \end{array} \right].$$

$$2A - 3B = 2 \left[\begin{array}{rr} 1 & -2 \\ 3 & 5 \end{array} \right] - 3 \left[\begin{array}{rr} -2 & -1 \\ 5 & 7 \end{array} \right] = \left[\begin{array}{rr} 2 & -4 \\ 6 & 10 \end{array} \right] - \left[\begin{array}{rr} -6 & -3 \\ 15 & 21 \end{array} \right] =$$

$$\left[\begin{array}{cc} 2 - (-6) & -4 - (-3) \\ 6 - 15 & 10 - 21 \end{array} \right] = \left[\begin{array}{rr} 8 & -1 \\ -9 & -11 \end{array} \right].$$

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