HOMEWORK 9: SOLUTIONS - MATH 111
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Problem 1 *Let A, B be sets in a universe U. Suppose that $n(A) = 3, n(B) = 9, n(A \cap B) = 1$ and $n(U) = 31$. what is $n((A \cup B)^c)$?*

Solution:

We have $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 3 + 9 - 1 = 11$. Therefore $n((A \cup B)^c) = n(U) - n(A \cup B) = 31 - 11 = 20$. ■

Problem 2 *Suppose that A, B and C are sets in a universe U. If $n(A) = 22, n(B) = 20, n(C) = 12, n(A \cap B) = 11, n(A \cap C) = 9, n(B \cap C) = 5, n(A \cap B \cap C) = 3$ and $n(U) = 44$, fill in the number of elements of each region in the appropriate Venn diagram.*

Solution:

We have

region	number
$A \cap B \cap C$	3
$A \cap B \cap C^c$	8
$A \cap B^c \cap C^c$	5
$A \cap B^c \cap C$	6
$A^c \cap B \cap C$	2
$A^c \cap B \cap C^c$	7
$A^c \cap B^c \cap C^c$	12
$A^c \cap B^c \cap C$	1

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Problem 3 *Consider the experiment of tossing a coin four times.*

(a) Write the sample space for this experiment.

(b) Give the event "an even number of heads occurs".

Solution:

(a) $S = \{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT\}$.

(b) $E = \{TTTT, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HHHH\}$. ■

Problem 4 *Consider the experiment of drawing a card from an ordinary deck of 52 cards. Find the probability of drawing a black face card.*

Solution:
Let E be the event of drawing a black face card. Then

$$P(E) = \frac{|E|}{|S|} = \frac{6}{52} = \frac{3}{26}.$$

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Problem 5 *Consider the experiment of successively drawing two balls out of an urn that contains 5 white, 6 red, 12 green and 7 black balls without replacement. What is the probability of the first ball being red and the second black? (This is one probability.)*

Solution:

$$\begin{aligned} P(1R \cap 2B) &= P(2B|1R)P(1R) \\ &= \tfrac{7}{29} \cdot \tfrac{6}{30} \\ &= \tfrac{7}{145}. \end{aligned}$$

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Problem 6 *Consider a very small community college with 500 students. This fall semester 150 students are taking Business 101 and 120 are taking Math 111. 70 of these students are taking both Business 101 and Math 111. If one of the students is picked at random from the college population, what is the probability that he will be taking Business 101 or Math 111?*

Solution:

$$\begin{aligned} P(B \cup M) &= P(B) + P(M) - P(B \cap M) \\ &= \tfrac{150}{500} + \tfrac{120}{500} - \tfrac{70}{500} \\ &= \tfrac{200}{500} \\ &= 0.4 \end{aligned}$$

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Problem 7 *Suppose that for two events E and F in a sample space S you know that $P(E) = 0.4, P(F^c) = 0.3$ and $P((E \cap F)^c) = 0.85$. Can you find $P((E \cup F)^c)$?*

Solution:
We have $P(F) = 1 - P(F^c) = 0.7$. Also $P(E \cap F) = 1 - P((E \cap F)^c) = 0.15$. Therefore $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.4 + 0.7 - 0.15 = 0.95$. Hence $P((E \cup F)^c) = 1 - P(E \cup F) = 1 - 0.95 = 0.05$. ■

Problem 8 *Consider the experiment of drawing successively two cards out of an ordinary deck of 52 cards without replacement. What is the probability of the second card being a red King given that the first card was a face card?*

Solution:
We have

$$\begin{aligned}
P(2RK|1F) &= \frac{P(2RK \cap 1F)}{P(1F)} \\
&= \frac{P(2RK \cap 1RK) + P(2RK \cap 1(F \cap RK^c)}{P(1F)} \\
&= \frac{P(2RK|1RK)P(1RK) + P(2RK|1(F \cap RK^c))P(1(F \cap RK^c))}{P(1F)} \\
&= \frac{\frac{1}{51}\frac{2}{52} + \frac{2}{51}\frac{10}{52}}{\frac{12}{52}} \\
&= \frac{\frac{22}{51 \cdot 52}}{\frac{12}{52}} \\
&= \frac{22}{12 \cdot 51} \\
&= \frac{11}{306}.
\end{aligned}$$

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