EXAM 1: SOLUTIONS - MATH 111
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Problem 1 *Find the slope of the line with equation* $5x + 3y = 21$.

Solution:

The secret is to solve for y and determine the coefficient of x in the slope-intercept form $y = ax + b$. We have $5x + 3y = 21$ implies $3y = 21 - 5x$, whence $y = -\frac{5}{3}x + 7$. Hence, the slope is $m = -\frac{5}{3}$. ■

Problem 2 *Find the equation of the line that is perpendicular to the line* $y = 7x + 2002$ *and passes through the point* $(1, \frac{6}{7})$.

Solution:

The given line has slope 7. Therefore the slope m of the unknown line is such that $7m = -1$. Thus $m = -\frac{1}{7}$. Applying now the point-slope form, we get $y - \frac{6}{7} = -\frac{1}{7}(x - 1)$, whence $y = -\frac{1}{7}x + 1$. ■

Problem 3 *The cost* C *in terms of the number of items* x *produced is given by* $C(x) = 3x + 10$ *and the revenue by* $R(x) = 5x$. *Find the revenue, when the company breaks even.*

Solution:

When the company breaks even $C(x) = R(x)$, whence $3x + 10 = 5x$, i.e., $2x = 10$ and, therefore $x = 5$. The break-even revenue is, thus, $R(5) = 25$. ■

Problem 4 *The supply* S *of an item in terms of the price* p *is given by* $S(p) = -p^2 + 300$ *and the demand* D *by* $D(p) = 20p$. *Determine the equilibrium price and the equilibrium supply.*

Solution:

At equilibrium $S(p) = D(p)$. Hence $-p^2 + 300 = 20p$, whence $p^2 + 20p - 300 = 0$. Use the quadratic formula to compute

$$p_{1,2} = \frac{-20 \pm \sqrt{(20^2 - 4 \cdot 1 \cdot (-300))}}{2 \cdot 1} = \frac{-20 \pm 40}{2} = -10 \pm 20.$$

Since p is the price we have $p = 10$. Thus, the equilibrium supply is $S(10) = -100 + 300 = 200$. ■

Problem 5 *Solve the inequality* $|x - 5| - 2 \leq 6$.

Solution:

$|x - 5| - 2 \leq 6$ implies $|x - 5| \leq 8$ which gives $-8 \leq x - 5 \leq 8$, whence $-3 \leq x \leq 13$. ■

Problem 6 *Find the domain of* $f(x) = \sqrt{\frac{x+2}{x-5}}$.

Solution:

We must have first $x - 5 \neq 0$, i.e., $x \neq 5$. Next, we also need $\frac{x+2}{x-5} \geq 0$, whence $(x+2)(x-5) \geq 0$. $x + 2 = 0$ gives $x = -2$ and $x - 5 = 0$ gives $x = 5$. Now create the sign table to conclude that $x \leq -2$ or $x \geq 5$. Because of the previous restriction that $x \neq 5$, we finally get

$$x \leq -2 \quad \text{or} \quad x > 5.$$

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