EXAM 4: SOLUTIONS - MATH 111
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Problem 1 *A polling of 100 voters in Germany over the German parties handling of the issue of war against terrorism revealed that 51 were supporting the social democrats, 50 the Christian democrats and 35 the greens. Of the 100 voters 22 said they were supporting both the social democrats and the Christian democrats, 16 were supporting both the Social democrats and the green and 18 both the Christian democrats and the green. Finally 5 voters expressed their support to all 3 major parties. How many voters did not support either of the three party positions?*

Solution:

The following table gives the numbers that fit the different regions of the Venn diagram, where S, C and G denote, respectively, Social democrats, Christian democrats and Greens:

region	number
$S \cap C \cap G$	5
$S \cap C \cap G^c$	17
$S \cap C^c \cap G^c$	18
$S \cap C^c \cap G$	11
$S^c \cap C \cap G$	13
$S^c \cap C \cap G^c$	15
$S^c \cap C^c \cap G^c$	15
$S^c \cap C^c \cap G$	6

Hence 15 did not support either party. ■

Problem 2 *A survey was conducted to explore the taste of two new beers A and B. 33% of the testers liked only beer B. 80% did not like A or did not like B. Finally 32% liked neither A nor B. Of a person selected at random among the testers, what is the probability that he/she likes beer A?*

Solution:

80% did not like A or did not like B. Therefore 20% liked both A and B. This gives $100 - 33 - 20 - 32 = 15$ that liked only A. Therefore $20 + 15 = 35$ percent liked A. Thus, the probability that someone selected at random likes A is 0.35. ■

Problem 3 *Three fair coins are tossed. Find*

1. *the probability of two heads and one tail appearing given that at least one head appeared.*
2. *the probability of two heads and one tail appearing given that at least one tail appeared.*

Solution:

1. We need the probability $P(E|F)$, where $F = \{TTH, THT, HTT, THH, HTH, HHT, HHH\}$ and $E = \{THH, HTH, HHT\}$. Thus, we have

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(\{THH, HTH, HHT\})}{P(F)} = \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}.$$

2. Similarly, we need the probability $P(E|G)$, where $G = \{TTT, TTH, THT, HTT, THH, HTH, HHT\}$ and $E = \{THH, HTH, HHT\}$. Thus, we have

$$P(E|F) = \frac{P(E \cap G)}{P(G)} = \frac{P(\{THH, HTH, HHT\})}{P(G)} = \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}.$$

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Problem 4 *A jar contains 6 red, 3 black, 7 white and 4 green balls. Two balls are drawn at random without replacement. Find the probability of*

1. *the first ball being white and the second being black,*
2. *one ball being red and one being green.*

Solution:

1. We have

$$\begin{aligned} P(1W \cap 2B) &= P(2B|1W) \cdot P(1W) \\ &= \tfrac{3}{19} \cdot \tfrac{7}{20} \\ &= \tfrac{21}{380}. \end{aligned}$$

2. We have

$$\begin{aligned} P(R \cap G) &= P(1R \cap 2G) + P(1G \cap 2R) \\ &= P(2G|1R) \cdot P(1R) + P(2R|1G) \cdot P(1G) \\ &= \tfrac{4}{19} \cdot \tfrac{6}{20} + \tfrac{6}{19} \cdot \tfrac{4}{20} \\ &= \tfrac{24}{380} + \tfrac{24}{380} \\ &= \tfrac{48}{380}. \end{aligned}$$

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Problem 5 *A section of college algebra is taught by Alex, Bob and Chris. 40% of the lectures are given by Alex, 35% by Bob and the remaining 25% by Chris. Chris, being the mathematically least talented, has 0.05 probability of committing an error during his presentations. Bob commits errors with probability 0.02, whereas, Alex has only 0.01 probability of committing an error during his own lectures. Given that in today's college algebra class an error was committed, what is the probability that Bob was giving the lecture?*

Solution:
We have

$$\begin{aligned} P(B|E) &= \frac{P(E|B)P(B)}{P(E|A)P(A)+P(E|B)P(B)+P(E|C)P(C)} \\ &= \frac{0.02 \cdot 0.35}{0.01 \cdot 0.4+0.02 \cdot 0.35+0.05 \cdot 0.25}. \end{aligned}$$

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Problem 6 *In a U.S. state, 20% of the population lives in inner cities, 35% in suburbs and 45% in rural areas. 20% of those living in inner cities receive poor medical care and the corresponding probabilities for those living in the suburbs and in rural areas are 5% and 10%, respectively. Find the probability that a person in the population selected at random receives satisfactory care.*

Solution:
Let F denote the event "satisfactory care", and I, S, R the events of coming from inner cities, suburbs and rural areas, respectively. Then

$$\begin{aligned} P(F) &= P(F \cap I) + P(F \cap S) + P(F \cap R) \\ &= P(F|I)P(I) + P(F|S)P(S) + P(F|R)P(R) \\ &= 0.8 \cdot 0.2 + 0.95 \cdot 0.35 + 0.9 \cdot 0.45. \end{aligned}$$

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