HOMEWORK 5: SOLUTIONS - MATH 111
INSTRUCTOR: George Voutsadakis

Problem 1 *Find the vertex, the opening direction, the intercepts and make a rough sketch of the graph of the function* $f(x) = -x^2 - 2x$.

Solution:

We have $V = (-\frac{b}{2a}, f(-\frac{b}{2a}))$. Therefore $x = -\frac{b}{2a} = -\frac{-2}{2(-1)} = -1$ and $f(-1) = -(-1)^2 - 2(-1) = -1 + 2 = 1$. The vertex thus is at $V = (-1, 1)$.

The parabola will open down because $a = -1 < 0$.

The y-intercepts are given by setting $x = 0$. We then get $y = 0$. Finally, the x-intercepts are obtained by setting $y = 0$. We find $-x^2 - 2x = 0$ implies $-x(x+2) = 0$, i.e., $x = -2$ or $x = 0$.

The rough sketch follows:

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Problem 2 *Do the same for the function* $g(x) = x^2 - x - 2$.

Solution:

We have $V = (-\frac{b}{2a}, f(-\frac{b}{2a}))$. Therefore $x = -\frac{b}{2a} = -\frac{-1}{2} = \frac{1}{2}$ and $f(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} - 2 = \frac{1}{4} - \frac{1}{2} - 2 = -\frac{9}{4}$. The vertex thus is at $V = (\frac{1}{2}, -\frac{9}{4})$.

The parabola will open up because $a = 1 > 0$.

The y-intercepts are given by setting $x = 0$. We then get $y = -2$. Finally, the x-intercepts are obtained by setting $y = 0$. We find $x^2 - x - 2 = 0$ implies $(x-2)(x+1) = 0$, i.e., $x = -1$ or $x = 2$.

The rough sketch follows:

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Problem 3 *Find the vertex and the opening direction of the graph of* $h(x) = (x-1)^2 - 3$.

Solution:

Compare with the form $h(x) = a(x-h)^2 + k$. We see that $a = 1 > 0$, whence the parabola opens up and that $h = 1, k = -3$, whence its vertex is at the point $V = (h, k) = (1, -3)$. ■

Problem 4 *Find the equation of the function whose graph is a parabola with vertex* $V = (1, 1)$ *passing through* $(-1, 0)$.

Solution:

The general formula of a parabola with vertex at (h, k) is given by $f(x) = a(x-h)^2 + k$. Since the vertex is at the point $V = (1, 1)$, the parabola will have the equation $f(x) = a(x-1)^2 + 1$. But we also know that it passes through the point $(-1, 0)$, whence we must have

$$0 = a(-1-1)^2 + 1, \quad \text{i.e.,} \quad 0 = (-2)^2 a + 1,$$

which gives $4a = -1$, and, hence, $a = -\frac{1}{4}$. Thus, the equation of the parabola is $f(x) = -\frac{1}{4}(x-1)^2 + 1$. ■

Problem 5 *When the price of a bizz is* $p(x) = 200 - x$, *then* x *bizz are sold. Find an expression for the revenue* $R(x)$ *in terms of the number* x *of bizz. Find the number of bizz that have to be sold to maximize the revenue and the maximum revenue.*

Solution:

The revenue is the product of the number of items sold times the price per item. Thus, it is given by $R(x) = xp(x) = x(200 - x) = -x^2 + 200x$. This is a parabola that opens down and, therefore, the maximum revenue will be achieved at its vertex point. The x-coordinate of the vertex is $-\frac{b}{2a} = -\frac{200}{2(-1)} = 100$ and the y-coordinate $y = R(-\frac{b}{2a}) = R(100) = 10,000$. Thus the maximum revenue is achieved when 100 bizz are sold and it is 10,000. ■

Problem 6 *An object is thrown upward with initial velocity 2 feet per second from an initial height of 3 feet. Then its height after* t *seconds is given by* $h(t) = -t^2 + 2t + 3$. *Find the maximum height that the object will attain and how long it will take for the object to hit the ground.*

Solution:

The height as a function of time is a parabola that opens down. Hence the maximum height will be achieved at the vertex of the parabola. This is given by $t = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$ and $h(1) = -1 + 2 + 3 = 4$. Hence the maximum height is 4 feet and it is reached after 1 second.

When the object hits the ground, its height will be 0. Hence $-t^2 + 2t + 3 = 0$, i.e., $t^2 - 2t - 3 = 0$, and, therefore $(t-3)(t+1) = 0$. This gives $t = -1$ or $t = 3$. Thus, the object will hit the ground after 3 seconds. ■

Problem 7 *Create the sign table and graph the function* $f(x) = 2x^3 - 3x^2$.

Solution:
These are shown below:

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Problem 8 *Find the horizontal and vertical asymptotes of the function* $g(x) = \frac{x+7}{x-3}$. *Then roughly sketch its graph.*

Solution:
The vertical asymptote is $x = 3$, i.e., at the zero of the denominator. The horizontal asymptote is at $y = \frac{1}{1} = 1$.

A rough sketch follows:

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