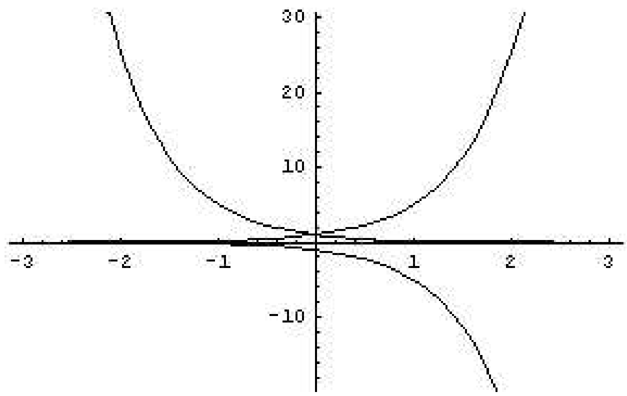

Figure 1: The graphs of the three exponentials.

HOMEWORK 6: SOLUTIONS - MATH 111
INSTRUCTOR: George Voutsadakis

Problem 1 *Find the equations of the vertical and horizontal asymptotes of the function* $f(x) = \frac{x^2-2x-3}{x^2-7x+10}$.

Solution:
We have
$$f(x) = \frac{x^2 - 2x - 3}{x^2 - 7x + 10} = \frac{(x-3)(x+1)}{(x-2)(x-5)}.$$
Thus, the roots of the numerator are $x = -1$ and $x = 3$ and the roots of the denominator are $x = 2$ and $x = 5$. This shows that the vertical asymptotes are the vertical lines $x = 2$ and $x = 5$. The horizontal asymptote is the line $y = \frac{a}{a'}$, where a and a', respectively, are the coefficients of the highest degree terms of the numerator and of the denominator, respectively. Thus, $y = \frac{1}{1} = 1$ is the horizontal asymptote. ■

Problem 2 *Graph on the same axis the functions* $f(x) = 5^x, g(x) = 5^{-x}$ *and* $h(x) = -5^x$. *Before graphing compute their values at* $x = 0$ *and* $x = 1$ *and depict those clearly on your graphs.*

Solution:
The tables of values for the three functions are shown below:

x	$y = 5^x$	$y = 5^{-x}$	$y = -5^x$
-1	$\frac{1}{5}$	5	$-\frac{1}{5}$
0	1	1	1
1	5	$\frac{1}{5}$	-5

Hence the three graphs are shown in Figure 1. ■

Problem 3 *Solve the equation* $5^{x^2} = 25^{2x-\frac{3}{2}}$.

Solution:

We have $5^{x^2} = (5^2)^{2x-\frac{3}{2}}$, whence $5^{x^2} = 5^{2(2x-\frac{3}{2})}$, and, therefore, $x^2 = 2(2x - \frac{3}{2})$. Hence $x^2 = 4x - 3$ and $x^2 - 4x + 3 = 0$. This gives $(x-3)(x-1) = 0$, whence $x = 1$ or $x = 3$. ■

Problem 4 *Solve the equation* $7^{-x+5} = (\frac{1}{7})^{2x-3}$.

Solution:

$7^{-x+5} = (7^{-1})^{2x-3}$, whence $7^{-x+5} = 7^{-(2x-3)}$, i.e., $-x+5 = -2x+3$, which yields $x = -2$. ■

Problem 5 *Culture studies in the lab have determined that the population of an organism A as a function of time t is given by $f(t) = e^{t^2-2t}$. At the same time, the population of another organism B in the same culture has been declining according to the function $g(t) = e^{-2t+1}$. At what time will the two organisms have the same populations in the culture?*

Solution:

We must have $f(t) = g(t)$, whence $e^{t^2-2t} = e^{-2t+1}$, and, hence, $t^2 - 2t = -2t + 1$, which yields $t^2 = 1$, and, thus, $t = \pm 1$. But t denotes time, whence $t = 1$. ■

Problem 6 *Compute* $\ln(\sqrt[6]{e})$ *and* $\ln(e^7)$ *without using a calculator.*

Solution:

$$\begin{aligned} \ln(\sqrt[6]{e}) &= \ln(e^{\frac{1}{6}}) \\ &= \tfrac{1}{6}\ln e \\ &= \tfrac{1}{6}. \end{aligned}$$

and

$$\begin{aligned} \ln(e^7) &= 7\ln e \\ &= 7. \end{aligned}$$

■

Problem 7 *If* $\ln x = 3$ *and* $\ln y = 4$ *find* $\ln(\sqrt{x} \cdot y^2)$.

Solution:

We have

$$\begin{aligned} \ln(\sqrt{x} \cdot y^2) &= \ln(\sqrt{x}) + \ln(y^2) \\ &= \ln(x^{\frac{1}{2}}) + \ln(y^2) \\ &= \tfrac{1}{2}\ln x + 2\ln y \\ &= \tfrac{1}{2} \cdot 3 + 2 \cdot 4 \\ &= \tfrac{3}{2} + 8 \\ &= \tfrac{19}{2}. \end{aligned}$$

■

Problem 8 *Solve the equation* $\log_2 x - \log_2 (x-3) = 3$.

Solution:
We get

$$\log_2 \frac{x}{x-3} = \log_2 8,$$

whence $\frac{x}{x-3} = 8$, which yields $x = 8(x-3)$, i.e., $x = 8x - 24$, and, therefore, $7x = 24$, or $x = \frac{24}{7}$. ■