HOMEWORK 7: SOLUTIONS - MATH 111
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Problem 1 *Solve the equation* $\log_{18} x + \log_{18} (x - 7) = 1$.

Solution:

We have $\log_{18} x + \log_{18} (x-7) = 1$ implies $\log_{18} (x(x-7)) = 1$, whence $x(x-7) = 18$. Therefore $x^2 - 7x - 18 = 0$, i.e., $(x-9)(x+2) = 0$. Thus, $x = -2$ or $x = 9$. But only $x = 9$ is an acceptable solution because for $x = -2$ both arguments of the logarithms $\log_{18} x$ and $\log_{18} (x-7)$ become negative. ■

Problem 2 *Solve the equation* $\log (x^3) = (\log x)^2$.

Solution:

$\log (x^3) = (\log x)^2$ implies $3 \log x = (\log x)^2$, whence $(\log x)^2 - 3 \log x = 0$, i.e.,

$$\log x(\log x - 3) = 0.$$

Hence $\log x = 0$ or $\log x - 3 = 0$. These give $\log x = 0$ or $\log x = 3$. Thus $x = 10^0$ or $x = 10^3$, whence $x = 1$ or $x = 1000$. ■

Problem 3 *The growth of an outpatient surgery as a percent of total surgeries at hospitals is approximated by* $f(x) = -1317 + 304 \ln x$, *where* x *represents the number of years since 1900.*

(a) What does this function predict for the percent of outpatient surgeries in 1998?

(b) When did outpatient surgeries reach 50%?

Solution:

(a) Since x represents the number of years since 1900, the percent of outpatient surgeries in 1998 is given by $f(98) = -1317 + 304 \ln 98$.

(b) We need to find x such that $f(x) = 50$ and then add 1900 to it to find the year. $f(x) = 50$ implies $-1317 + 304 \ln x = 50$, i.e., $304 \ln x = 1367$, whence $\ln x = \frac{1367}{304}$, and, therefore, $x = e^{\frac{1367}{304}}$. Thus the answer for the year would be $1900 + e^{\frac{1367}{304}}$. ■

Problem 4 *Find the simple interest on a loan of $40,000 at 6% made on September 1 and due on November 30.*

Solution:

We have to use the simple interest formula

$$I = Prt,$$

where $P = 40,000, r = 0.06$ and $t = 0.25$. Thus $I = 40,000 \cdot 0.06 \cdot 0.25 = 10,000 \cdot 0.06 = 600$. ■

Problem 5 *A friend of yours decided to go back to college. She decides to buy a small car for \$6,000. She intends to borrow the money from a bank with 10% discount rate. If she plans to repay the loan in 2 years what will be the amount of her loan?*

Solution:
Recall the discount formula

$$P = A(1 - rt).$$

We have $P = 6000, r = 0.1$ and $t = 2$ and we need to solve for A. Thus $A = \frac{P}{1-rt}$, whence $A = \frac{6000}{1-0.1\cdot 2} = \frac{6000}{0.8} = 7500$. ■

Problem 6 *Find the amount of interest earned by a deposit of \$10,000 compounded semi-annually at 5% for 3 years.*

Solution:
We have that

$$I = P(1 + \frac{r}{m})^{tm} - P,$$

where $P = 10000, r = 0.05, m = 2$ and $t = 3$. Hence $I = 10000(1 + \frac{0.05}{2})^{2\cdot 3} - 1000$, i.e., $I = 10000(1.025)^6 - 10000 = 1596.934$. ■

Problem 7 *Find the present value of the future amount \$5,000 compounded semiannually at 3% for 2 years.*

Solution:
We have

$$A = P(1 + \frac{r}{m})^{tm},$$

where $A = 5000, r = 0.03, m = 2$ and $t = 2$. Hence $P = \frac{A}{(1+\frac{r}{m})^{tm}} = \frac{5000}{(1.015)^4} = 4710.92$. ■

Problem 8 *Find the sum of the first four terms of the geometric sequence with first term $a = 2$ and common ratio $r = 3$.*

Solution:
We have

$$S_n = a\frac{1 - r^n}{1 - r},$$

whence for $n = 4$,

$$S_4 = 2\frac{1 - 3^4}{1 - 3} = 2\frac{-80}{-2} = 80.$$

■