HOMEWORK 8: SOLUTIONS - MATH 111
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Problem 1 *Solve the systems*

$$\left\{\begin{array}{rcrcr} 3x & + & 5y & = & 12 \\ -x & + & 2y & = & 7 \end{array}\right\}, \quad \left\{\begin{array}{rcrcr} 2x & - & 3y & = & 1 \\ 8x & - & 12y & = & -4 \end{array}\right\},$$

by the substitution method.

Solution:

For the first system we have, solving the second equation for x, $x = 2y - 7$. Substituting this value for x into the first equation. we get $3(2y-7)+5y = 12$, i.e., $6y - 21 + 5y = 12$. Therefore $11y = 33$, which gives $y = 3$. Thus $x = 6 - 7 = -1$. The solution, thus, is $(x, y) = (-1, 3)$.

For the second system, solving the first equation for x, $x = \frac{3}{2}y + \frac{1}{2}$. Substituting this value into the second equation one obtains $8(\frac{3}{2}y+\frac{1}{2})-12y = -4$, whence $12y+4-12y = -4$. But this yields $4 = -4$ and therefore the system is inconsistent, i.e., it does not have any solutions. ∎

Problem 2 *Solve the system* $\left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & -6 \\ -x & + & 2y & + & z & = & 8 \\ 2x & - & 3y & + & 2z & = & -10 \end{array}\right\}$ *by using allowable operations on the equations (Gauss elimination).*

Solution:

We have

$$\left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & -6 \\ -x & + & 2y & + & z & = & 8 \\ 2x & - & 3y & + & 2z & = & -10 \end{array}\right\} \longrightarrow \left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & -6 \\ & & & & 2z & = & 2 \\ & & y & & & = & 2 \end{array}\right\} \longrightarrow$$

$$\left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & -6 \\ & & & & z & = & 1 \\ & & y & & & = & 2 \end{array}\right\} \longrightarrow \left\{\begin{array}{rcrcrcr} x & & & + & z & = & -2 \\ & & & & z & = & 1 \\ & & y & & & = & 2 \end{array}\right\} \longrightarrow$$

$$\left\{\begin{array}{rcrcrcr} x & & & & & = & -3 \\ & & & & z & = & 1 \\ & & y & & & = & 2 \end{array}\right\}$$

Hence $(x, y, z) = (-3, 2, 1)$. ∎

Problem 3 *Solve the system* $\left\{\begin{array}{rcrcrcr} x & + & y & + & z & = & -2 \\ -x & - & 2y & + & 3z & = & 1 \\ 2x & + & y & - & 2z & = & 3 \end{array}\right\}$ *by using the Gauss-Jordan method (matrix row operations).*

Solution:
We have

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & -2 \\ -1 & -2 & 3 & 1 \\ 2 & 1 & -2 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 1 & 1 & -2 \\ 0 & -1 & 4 & -1 \\ 0 & -1 & -4 & 7 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 1 & 1 & -2 \\ 0 & 1 & -4 & 1 \\ 0 & -1 & -4 & 7 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 5 & -3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & -8 & 8 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & 5 & -3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Hence $(x, y, z) = (2, -3, -1)$. ■

Problem 4 *Solve the system* $\left\{\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 1 \\ -3x & - & y & + & z & = & 1 \\ 2x & + & 4y & - & 2z & = & 2 \end{array}\right\}$ *by using the Gauss-Jordan method (matrix row operations).*

Solution:

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ -3 & -1 & 1 & 1 \\ 2 & 4 & -2 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ 0 & 5 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{2}{5} & \frac{4}{5} \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & -\frac{1}{5} & -\frac{3}{5} \\ 0 & 1 & -\frac{2}{5} & \frac{4}{5} \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Thus, we get $(x, y, z) = (\frac{1}{5}z - \frac{3}{5}, \frac{2}{5}z + \frac{4}{5}, z)$, z any real number. ■

Problem 5 *Let* $A = \left[\begin{array}{rr} 1 & -2 \\ 3 & -1 \end{array}\right]$ *and* $B = \left[\begin{array}{rr} 3 & 6 \\ -8 & 5 \end{array}\right]$. *Compute* $A+B$, $A-B$ *and* $3A-2B$.

Solution:

$$A + B = \left[\begin{array}{rr} 1 & -2 \\ 3 & -1 \end{array}\right] + \left[\begin{array}{rr} 3 & 6 \\ -8 & 5 \end{array}\right] = \left[\begin{array}{rr} 4 & 4 \\ -5 & 4 \end{array}\right].$$

$$A - B = \left[\begin{array}{rr} 1 & -2 \\ 3 & -1 \end{array}\right] - \left[\begin{array}{rr} 3 & 6 \\ -8 & 5 \end{array}\right] = \left[\begin{array}{rr} -2 & -8 \\ 11 & -6 \end{array}\right].$$

$$3A - 2B = 3\left[\begin{array}{rr} 1 & -2 \\ 3 & -1 \end{array}\right] - 2\left[\begin{array}{rr} 3 & 6 \\ -8 & 5 \end{array}\right] = \left[\begin{array}{rr} 3 & -6 \\ 9 & -3 \end{array}\right] - \left[\begin{array}{rr} 6 & 12 \\ -16 & 10 \end{array}\right] = \left[\begin{array}{rr} -3 & -18 \\ 25 & -13 \end{array}\right].$$

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Problem 6 *Let* $A = \left[\begin{array}{rrr} 1 & 2 & 3 \\ -1 & 5 & 7 \end{array}\right]$ *and* $B = \left[\begin{array}{rrr} -10 & 2 & -7 \\ 3 & 2 & 0 \end{array}\right]$. *Compute* $A-B$ *and* $-2A+5B$.

Solution:

$$A - B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 7 \end{bmatrix} - \begin{bmatrix} -10 & 2 & -7 \\ 3 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 10 \\ -4 & 3 & 7 \end{bmatrix}.$$

$$-2A + 5B = -2\begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 7 \end{bmatrix} + 5\begin{bmatrix} -10 & 2 & -7 \\ 3 & 2 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} -2 & -4 & -6 \\ 2 & -10 & -14 \end{bmatrix} + \begin{bmatrix} -50 & 10 & -35 \\ 15 & 10 & 0 \end{bmatrix} = \begin{bmatrix} -52 & 6 & -41 \\ 17 & 0 & -14 \end{bmatrix}.$$

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Problem 7 *Let* $A = \begin{bmatrix} -1 & 3 & -5 & 0 \\ 2 & 6 & -1 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 0 \\ -1 & 7 & -3 \\ 1 & -5 & 9 \end{bmatrix}$. *Compute* $A \cdot B$ *and* $B \cdot A$.

Solution:

A is a 2×4 matrix and B is a 4×3 matrix. Therefore $A \cdot B$ is well-defined and the result is a 2×3 matrix. $B \cdot A$, however cannot be computed, because the number of columns of B is different from the number of rows of A. We have

$$A \cdot B = \begin{bmatrix} -1 & 3 & -5 & 0 \\ 2 & 6 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 0 \\ -1 & 7 & -3 \\ 1 & -5 & 9 \end{bmatrix} =$$

$$\begin{bmatrix} -1+9+5+0 & 0+6-35+0 & 2+0+15+0 \\ 2+18+1+1 & 0+12-7-5 & -4+0+3+9 \end{bmatrix} = \begin{bmatrix} 13 & -29 & 17 \\ 22 & 0 & 8 \end{bmatrix}.$$

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Problem 8 *Compute the inverses of the matrices* $A = \begin{bmatrix} -1 & 2 \\ 1 & -3 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 & -1 \\ 6 & -3 \end{bmatrix}$.

Solution:

We have $\begin{vmatrix} -1 & 2 \\ 1 & -3 \end{vmatrix} = 3 - 2 = 1 \neq 0$. Therefore $A^{-1} = \begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix}$.

Also $\begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} = -6 + 6 = 0$, whence B is not invertible. ■