EXAM 4: SOLUTIONS - MATH 111
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Problem 1 *Let* $U = \{2,3,4,5,7,9\}, X = \{2,3,4,5\}, Y = \{3,5,7,9\}$ *and* $Z = \{2,4,5,7,9\}$. *Compute the sets (a)* $X \cap Y$, *(b)* $X' \cup Y$ *and (c)* $Y \cap (X' \cup Z)$.

Solution:

$$\begin{array}{lcl} X \cap Y & = & \{2,3,4,5\} \cap \{3,5,7,9\} = \{3,5\}. \\ X' \cup Y & = & \{2,3,4,5\}' \cup \{3,5,7,9\} = \{7,9\} \cup \{3,5,7,9\} = \{3,5,7,9\}. \\ Y \cap (X' \cup Z) & = & \{3,5,7,9\} \cap (\{2,3,4,5\}' \cup \{2,4,5,7,9\}) \\ & = & \{3,5,7,9\} \cap (\{7,9\} \cup \{2,4,5,7,9\}) = \{3,5,7,9\} \cap \{2,4,5,7,9\} \\ & = & \{5,7,9\}. \end{array}$$

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Problem 2 *Human blood can contain either no antigens, the A antigen, the B antigen, or both the A and the B antigens. A third antigen, called the Rh antigen, is important in human reproduction, and again may or may not be present in an individual. Blood is called type A-positive if the individual has the A and Rh antigens, but not the antigen B. A person having only the A and B antigens is said to have type AB-negative blood. A person having only the Rh antigen has type O-positive blood. Other blood types are defined in a similar manner.*

In the Soo hospital, the following data were recorded: 25 patients had the A antigen, 17 had the A and B antigens, 27 had the B antigen, 22 had the B and Rh antigens, 30 had the Rh antigen, 12 had none of the antigens, 16 had the A and Rh antigens and 15 had all three antigens. Find
(a) How many patients were represented
(b) How many had exactly two antigens
(c) How many had O-negative blood type.

Solution:

Let A denote the set of those patients having the A antigen, B the B antigen and R the Rh antigen. Then the different areas of the Venn diagram are filled as follows:

Area	Number
$A \cap B \cap R$	15
$A \cap B \cap R'$	2
$A \cap B' \cap R$	1
$A' \cap B \cap R$	7
$A \cap B' \cap R'$	7
$A' \cap B \cap R'$	3
$A' \cap B' \cap R$	7
$A' \cap B' \cap R'$	12

Hence 54 patients were represented, 10 had exactly two antigens and 12 had O-negative blood type. ■

Problem 3 *Consider the following experiment: A fair coin is tossed. If it shows heads, then a fair die is rolled. If tails shows, the experiment is ended.*
(a) Write a sample space for this experiment.
(b) What is the probability of the event "Tails"?
(c) What is the probability of the event "An even number was rolled given that heads was tossed"?
(d) What is the probability of the event "Heads was tossed and an even number was rolled"?

Solution:

(a) $S = \{H1, H2, H3, H4, H5, H6, T\}$.

(b) $P(T) = \frac{1}{2}$.

(c) $P(\text{EVEN}|\text{H}) = \frac{1}{2}$.

(d) $P(\text{EVEN} \cap H) = P(\text{EVEN}|\text{1H})\text{P(H)} = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$.

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Problem 4 *Sixty students in a Detroit school were interviewed with the following results: 35 spoke Spanish, 15 spoke Chinese and 6 spoke both languages. Find the probability that a randomly selected student from this school (a) speaks neither of these languages*
(b) speaks only one of the two languages.

Solution:

If the Venn diagram is filled-in correctly, we obtain that 16 students did not speak either language and 38 spoke only one of the two languages. Therefore the probability that a randomly selected student from this school speaks neither language is $\frac{16}{60} = \frac{4}{15}$ and the probability that he/she speaks only one of the two languages is $\frac{38}{60} = \frac{19}{30}$. ■

Problem 5 *Consider the experiment of drawing successively two cards from a well-shuffled deck without repetition. Find the probability that the second card drawn is red.*

Solution:

$$\begin{aligned}
P(2ndR) &= P((2ndR \cap 1stR) \cup (2ndR \cap 1stB)) \\
&= P(2ndR \cap 1stR) + P(2ndR \cap 1stB) \\
&= P(2ndR|1stR)P(1stR) + P(2ndR|1stB)P(1stB) \\
&= \frac{25}{51}\frac{1}{2} + \frac{26}{51}\frac{1}{2} \\
&= (\frac{25}{51} + \frac{26}{51})\frac{1}{2} \\
&= 1 \cdot \frac{1}{2} \\
&= \frac{1}{2}.
\end{aligned}$$

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Problem 6 *Consider the experiment of drawing successively two cards from a well-shuffled deck without repetition. Find the probability that the second card drawn is red.*

The Soo-Coop Bank finds that the relationship between mortgage defaults and the size of the down payment is given by the following table

Down Payment (%)	10%	20%	25%
Number of mortgages of this type	300	200	100
Probability of default	0.03	0.02	0.01

What is the probability that a default will occur? If a default occurs, what is the probability that it occurred on a mortgage with a 25% down payment?

Solution:

$$\begin{aligned} P(D) &= P(D|10)P(10) + P(D|20)P(20) + P(D|25)P(25) \\ &= \tfrac{1}{2}0.03 + \tfrac{1}{3}0.02 + \tfrac{1}{6}0.01 \\ &= \tfrac{14}{600} = 0.0233. \end{aligned}$$

$$P(25|D) = \frac{P(D|25)P(25)}{P(D)} = \frac{\frac{1}{6}0.01}{\frac{14}{600}} = \frac{1}{14} = 0.0714.$$

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