HOMEWORK 3: SOLUTIONS - MATH 111
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Problem 1 *The Revenue R in terms of the number of items produced is given by $R(x) = 12x$ and the cost C by $C(x) = 7x + 85$. Find the break-even point and the break-even price.*

Solution:
At the break-even point we have $R(x) = C(x)$. Hence $12x = 7x + 85$, which yields $5x = 85$, and therefore $x = 17$. The break-even revenue is thus $R(17) = 12 \cdot 17 = 204$. ■

Problem 2 *The supply S and the demand D in terms of the number of items q are given by $S(q) = \frac{1}{5}q + 5$ and $D(q) = -q + 47$, respectively. Find the equilibrium demand and the equilibrium price.*

Solution:
At the equilibrium point $S(q) = D(q)$. Thus, $\frac{1}{5}q + 5 = -q + 47$, whence $\frac{6}{5}q = 42$, which yields $q = 35$. The equilibrium price is $D(35) = S(35) = -35 + 47 = 12$. ■

Problem 3 *Find the number of solutions of $5x^2 - 6x + 2 = 0$.*

Solution:
For the number of solutions of a quadratic one only has to compute the discriminant $D = b^2 - 4ac$ and check its sign. If $D > 0$, then the quadratic has two different solutions. If $D = 0$, then the quadratic has one double root and if $D < 0$, then the quadratic does not have any real roots. In the present case we have $D = b^2 - 4ac = (-6)^2 - 4 \cdot 5 \cdot 2 = 36 - 40 = -4 < 0$. Hence the quadratic does not have any real roots. ■

Problem 4 *Use the quadratic formula to solve $7x^2 + 2x - 3 = 0$.*

Solution:
Compute the discriminant $D = b^2 - 4ac = 2^2 - 4 \cdot 7 \cdot (-3) = 4 + 84 = 88$. Hence $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2 \pm \sqrt{88}}{2 \cdot 7} = \frac{-2 \pm 2\sqrt{22}}{14} = \frac{-1 \pm \sqrt{22}}{7}$, whence $x_1 = \frac{-1-\sqrt{22}}{7}$ and $x_2 = -\frac{-1+\sqrt{22}}{7}$. ■

Problem 5 *Solve the inequality $x^2 + 4x - 18 \geq 3$.*

Solution:
First subtract 3 from both sides to obtain $x^2 + 4x - 21 \geq 0$. The left hand side now factors as $(x+7)(x-3) \geq 0$. Therefore, by building the sign table one discovers that $x \leq -7$ or $x \geq 3$ give the solutions for this inequality. ■

Problem 6 *Solve the inequality $\frac{x-11}{x+25} \leq 0$.*

Solution:
This inequality is also solved by constructing the sign table for the fraction $\frac{x-11}{x+25}$. One then sees that the fraction becomes ≤ 0 when $-25 < x \leq 11$. ■

Problem 7 *Find the domain of* $f(x) = |5x + 9|$.

Solution:

Since no denominators or square roots appear in the expression defining $f(x)$ the domain of f is the set $D(f) = \mathbb{R} = (-\infty, +\infty)$ of all real numbers. ■

Problem 8 *Find the domain of* $g(x) = \sqrt{\frac{x-5}{x^2+2x-3}}$.

Solution:

The two restrictions that x should obey are, first, that $x^2 + 2x - 3 \neq 0$ and, second, that $\frac{x-5}{x^2+2x-3} \geq 0$. To find the x's that obey both we may set up the sign table for the fraction $\frac{x-5}{x^2+2x-3} = \frac{x-5}{(x+3)(x-1)}$. The sign table, if built correctly, would give us $-3 < x < 1$ or $x \geq 5$ as the range of values that satisfy both restrictions simultaneously. Hence $D(g) = \{x : -3 < x < 1 \text{ or } x \geq 5\}$. ■