HOMEWORK 4 - MATH 215

DUE DATE: After Chapter 2 has been covered!
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Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Prove or find a counterexample for each:
   (a) If A and B are inductive, then $A \cup B$ is inductive.
   (b) If A and B are inductive, then $A \cap B$ is inductive.

2. Give an inductive definition for each:
   (a) A set formed as an arithmetic progression $\{a, a+d, a+2d, \ldots\}$.
   (b) A set formed as a geometric progression $\{a, ar, ar^2, \ldots\}$.
   (c) $\bigcup_{i=1}^{n} A_i$, for some indexed family $\{A_i : i \in \mathbb{N}\}$.
   (d) The product $\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot \ldots \cdot x_n$ of n real numbers.

3. Use the PMI to prove the following for all natural numbers n.
   (a) $1 + 4 + 7 + \ldots + (3n-2) = \frac{1}{2}n(3n-1)$.
   (b) $n^3 + 5n + 6$ is divisible by 3.
   (c) $10^n + 3 \cdot 4^{n+2} + 5$ is divisible by 9.
   (d) $\prod_{i=1}^{n}(2i-1) = \frac{(2n)!}{n!2^n}$.
   (e) Using the differentiation formulas $\frac{d}{dx}(x) = 1$ and $\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$, prove that for all $n \in \mathbb{N}$, $\frac{d}{dx}(x^n) = nx^{n-1}$.

4. Use the generalized PMI to prove the following:
   (a) $(n+1)! > 2^{n+3}$ for $n \geq 5$.
   (b) For all $n > 2$, the sum of the angle measures of the interior angles of a convex polygon of n sides is $(n-2) \cdot 180^o$.

5. Suppose that a statement $P(n)$ satisfies:
   (a) $P(1)$ is true.
   (b) if $P(n)$ is true, then $P(n+2)$ is true.
   Is $P(n)$ true for all $n \in \mathbb{N}$? Explain.

6. Let $a_1 = 2, a_2 = 4$ and $a_{n+2} = 5a_{n+1} - 6a_n$ for all $n \geq 3$. Prove that $a_n = 2^n$ for all natural numbers n.

7. Let the "Foobar-nacci" numbers g_n be defined as follows: $g_1 = 2, g_2 = 2$ and $g_{n+2} = g_{n+1}g_n$, for all $n \geq 1$.
   (a) Calculate the first five "Foorbar-nacci" numbers.
   (b) Show that $g_n = 2^{f_n}$.

8. Find (a) $\#\{n \in \mathbf{Z} : n^2 < 41\}$ (b) $\#\{n \in \mathbb{N} : n + 1 = 4n - 10\}$

9. Of the four teams in a softball league, one team has four pitchers and the other teams have three each. Give the counting rules that apply to determine each of the following.
   (a) the number of possible selections of pitchers for an all-star team, if exactly four pitchers are to be chosen.
   (b) The number of possible selections if one pitcher is to be chosen from each team.
   (c) The number of possible selections of four pitchers, if exactly two of the five left-handed pitchers in the league must be selected.
   (d) The number of possible orders in which the four pitchers, once they are selected, can appear (one at a time) in the all-star game.

10. Among the 40 first-time campers at Camp Forlorn one week, 14 fell into the lake during the week, 13 suffered from poison ivy, and 16 got lost trying to find the dining hall. Three of these campers had poison ivy rash and fell into the lake, 5 fell into the lake and got lost, 8 had poison ivy and got lost and 2 experienced all three misfortunes. How many first-time campers got through the week without any of these mishaps?

11. Find the number of ways seven school children can line up to board a school bus.

12. Suppose the seven children of the previous exercise are three girls and four boys. Find the number of ways they could line up subject to these conditions.
    (a) The three girls are first in line.
    (b) The three girls are together in line.
    (c) The four boys are together in line.
    (d) No two boys are together.

13. Among ten lottery finalists, four will be selected to win individual amounts of $\$1,000, \$2,000$, $\$5,000$ and $\$10,000$. In how many ways may the money be distributed?

14. From a second-grade class of 11 boys and 8 girls, 3 are selected for flag duty.
    (a) How many selections are possible?
    (b) How many of these selections have exactly 2 boys?
    (c) Exactly 1 boy?

15. Prove combinatorially that if n is odd, then the number of ways to select an even number of objects from n is equal to the number of ways to select an odd number of objects.