HOMEWORK 6 - MATH 215

DUE DATE: After Chapter 3 has been covered!
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Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Which of the relations on the given sets are antisymmetric?
   (a) $A = \{1, 2, 3, 4, 5\}, R = \{(1, 3), (1, 1), (2, 4), (3, 2), (5, 4), (4, 2)\}$
   (b) $\mathbb{R}, xRy$ iff $x^2 = y^2$.
   (c) $A = \{1, 2, 3, 4\}$ and R is given by the following digraph.

2. Show that if R is antisymmetric, then xRy and $x \neq y$ implies $y \not R x$.

3. Give an example of a relation R on a set A which is antisymmetric and such that xRx for some, but not all, $x \in A$.

4. Give an example of a relation S on a set $A = \{a, b, c, d\}$ such that S is transitive, antisymmetric and irreflexive (that is, xRx is false for all $x \in A$).

5. Define the relation on $\mathbb{R} \times \mathbb{R}$ by $(a, b)R(x, y)$ iff $a \leq x$ and $b \leq y$. Prove that R is a partial ordering for $\mathbb{R} \times \mathbb{R}$.

6. Let $\mathbf{C}$ be the complex numbers. Define $(a + bi)R(c + di)$ iff $a^2 + b^2 \leq c^2 + d^2$. Is R a partial order for $\mathbf{C}$? Justify your answer.

7. Use your own judgement about which tasks should precede others to draw a Hasse diagram for the partial order among the tasks for the following complex job:
   To back a car out of the garage, Kim must perform 11 tasks:
   t_1: put the key in the ignition
   t_2:step on the gas
   t_3: check to see if the driveway is clear
   t_4:start the car
   t_5:adjust the mirror
   t_6:open the garage door t_7:fasten the seat belt
   t_8:adjust the position of the driver's seat
   t_9:get in the car
   t_{10}:put the car in reverse gear
   t_{11}: step on the break.

8. Let A be a set. Consider the partial order $\subseteq$ on $\mathcal{P}(A)$.
(a) Let C and D be subsets of A. Prove that the least upper bound of $\{C, D\}$ is $C \cup D$ and the greatest lower bound of $\{C, D\}$ is $C \cap D$.
(b) Let $\mathcal{B}$ be a family of subsets of A. Prove that the least upper bound of $\mathcal{B}$ is $\bigcup_{B \in \mathcal{B}} B$ and the greatest lower bound of $\mathcal{B}$ is $\bigcap_{B \in \mathcal{B}} B$.

9. For what sets A is $\mathcal{P}(A)$ with set inclusion a linear ordering?

10. Prove that every subset of a well-ordered set is well-ordered.

11. P is a preorder for a set A if P is reflexive and transitive relation on A. Define a relation E on A by xEy iff xPy and yPx. Show that E is an equivalence relation on A.

12. If possible, give an example of a graph with order 6 such that
(a) the vertices have degrees $1, 1, 1, 1, 1, 5$
(b) the vertices have degrees $1, 1, 1, 1, 1, 1$
(c) the vertices have degrees $2, 2, 2, 2, 2, 2$
(d) the vertices have degrees $1, 2, 2, 2, 3, 3$
(e) exactly two vertices have even degree.
(f) exactly two vertices have odd degree.

13. f possible give an example of a graph
(a) with order 6 and size 6
(b) with order 4 and size 6
(c) with order 3 and size 6
(d) with order 6 and size 3.

14. The **complement** $\tilde{G}$ of a graph $G(V, E)$ is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G. Give the complements of these graphs.

15. Give an example of a graph with 6 vertices having degrees $1, 1, 2, 2, 2, 2$ that is
(a) connected (b) disconnected

16. Give an example of a graph with 6 vertices having
(a) one component (b) two components (c) three components (d) six components.

17. Give an example of a graph with order 6 such that
(a) two vertices u and v have distance 5
(b) for any two vertices u and v, $d(u, v) \leq 2$.