EXAM 2 - MATH 325

Thursday, November 20, 2003
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Read each problem very carefully before starting to solve it. Each question is worth 4 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) State and prove the ASA Theorem for similar triangles

   (b) Let $\triangle ABC$ be a triangle and A' the midpoint of $\overline{BC}$. Let l be a line through A' that intersects $\overline{AB}$ and $\overline{AC}$ at D and E, respectively. Prove that $\frac{AB}{BD} = \frac{AE}{EC}$.
   (**Hint:** Depending on how you draw l, you may need to bring the parallel to $\overline{AC}$ through B or the parallel to $\overline{AB}$ through C. Feel free to consider only one of the possible cases!)

2. (a) Give the definition of a tangent line to a circle. Show that, if l is tangent to (O, r) at A, then l is perpendicular to $\overline{OA}$ at A. Use this to show that two tangents to (O, r) through the same point P are equal in length.

   (b) Show that if $\widehat{ABC}$ is inscribed in a circle (O, r), such that $\overline{AB}$ is a diameter, then $\widehat{ABC} = \frac{1}{2}\widehat{AOC}$.

3. (a) Let $\overline{AB}, \overline{CD}$ be two chords of a circle (O, r), intersecting at an interior point E. Prove that $AE \cdot EB = CE \cdot ED$.

   (b) Let $\triangle ABC$ be a triangle and $\overline{AD}$ the angle bisector of $\widehat{A}$ that intersects the circumcircle at E. Prove that $\frac{AB}{AE} = \frac{AD}{AC}$.

4. (a) Let $ABCD$ be a parallelogram. Let l be a line through A that intersects $\overline{BD}$ at E, $\overline{BC}$ at Z and $\overline{CD}$ at H. Show that $EA^2 = EZ \cdot EH$.

   (b) Prove that a regular n-gon inscribed in a circle of radius r and with sides of length s has area $\frac{n}{2}s\sqrt{r^2 - \frac{1}{4}s^2}$.

5. (a) Prove that a point P is on the perpendicular bisector of a line segment $\overline{AB}$ if and only if $PA = PB$. Use this to show that the perpendicular bisectors of the three sides of a triangle $\triangle ABC$ are concurrent.

   (b) Prove that in a triangle $\triangle ABC$, $bc = 2Rh_a$. Use this to derive Brahmagupta's Formula for the area K of the triangle.