HOMEWORK 2 - MATH 325

DUE DATE: When Chapter 3 has been covered!
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Given triangles $\triangle ABC$ and $\triangle DEF$ such that $\overline{AB}$ and $\overline{DE}$ are parallel and congruent and $\overline{BC}$ and $\overline{EF}$ are parallel and congruent, prove that $\overline{AC}$ and $\overline{DF}$ are parallel and congruent.

2. Given a quadrilateral $ABCD$ such that $\overline{AB} \parallel \overline{CD}$, prove that $\widehat{C} = \widehat{D}$ if and only if $\overline{AD} \cong \overline{BC}$.

3. Our proof that parallel lines are at a constant distance apart used the parallel postulate. Prove the following without using the parallel postulate: Assume that the lines l_1 and l_2 are at a constant distance apart. Then, if l_1 and l_2 are cut by a transversal, alternate interior angles must be congruent.

4. Let $ABCD$ be a parallelogram. Define the base to be $\overline{AB}$ and the height to be the distance between $\overline{AB}$ and $\overline{CD}$. Prove that $ABCD$ has area = base $\times$ height.

5. Assume that in quadrilateral $ABCD$ $\overline{AB} \parallel \overline{CD}$. Let $AB = b_1, CD = b_2$ and let h be the distance between $\overline{AB}$ and $\overline{CD}$. Prove that $ABCD$ has area $\frac{1}{2}h(b_1 + b_2)$.

6. Prove that in a right triangle, if the hypotenuse is the base of length c, then the height is $h = \frac{ab}{c}$.

7. Given a convex quadrilateral $ABCD$ with $AC \perp BD$, prove that $AB^2 + CD^2 = BC^2 + AD^2$.

8. (a) Given $\overline{AB}$ and $\overline{CD}$ construct $\overline{EF}$ such that $EF^2 = AB^2 + CD^2$.
   (b) Given $\overline{AB}, \overline{CD}$ and $\overline{EF}$, construct $\overline{GH}$ such that $GH^2 = AB^2 + CD^2 + EF^2$.

9. (a) Assume that in $\triangle ABC$, $a = 4, b = 9$ and $c = 11$. Calculate the area.
   (b) Calculate the length of each of the three altitudes in the triangle of part (a).
   (c) If we let $a = 2, b = 3$ and $c = 7$ in Heron's formula we get a problem. What is the problem and why does it happen?