HOMEWORK 6 - MATH 325

DUE DATE: After Chapter 9 has been covered!
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. In triangle $\triangle ABC$ let C_1 on $\overline{AB}$ be such that $AC_1 = \frac{1}{3}AB$ and let B_1 on $\overline{AC}$ be such that $AB_1 = \frac{1}{3}AC$. Prove:
   (a) $\overline{B_1C_1} \parallel \overline{BC}$ (b) $B_1C_1 = \frac{1}{3}BC$
   (c) If $\overline{BB_1}$ and $\overline{CC_1}$ intersect at point G_1, then $B_1G_1 = \frac{1}{4}BB_1$ and $C_1G_1 = \frac{1}{4}CC_1$.
2. Use the triangle inequality on $\triangle ABA'$ and $\triangle ACA'$ to prove that $2m_a > b + c - a$. What does this imply about $m_a + m_b + m_c$?
3. Given points B and C and a length k, prove that the set of points in the plane $\{A : AB^2 + AC^2 = k^2\}$ is a circle whose center is the midpoint of $\overline{BC}$.
4. What is the anticomplementary triangle of the complementary triangle of $\triangle ABC$?
5. Prove that if $\triangle ABC$ has complementary triangle $\triangle A'B'C'$ with centroid G, then the six triangles $\triangle GA'C, \triangle GCB', \triangle GB'A, \triangle GAC', \triangle GC'B$ and $\triangle GBA'$ all have equal areas.
6. In $\triangle ABC$ let $a = BC, b = AC, c = AB$. Assume that A_1 is a trisection point of $\overline{BC}$ with $A_1B = \frac{1}{3}a$. Prove that $3AA_1^2 = b^2 + 2c^2 - \frac{2}{3}a^2$.
7. Our construction of a line l through P and the inaccessible intersection point of l_1 and l_2 will not work if $l_1 \perp l_2$. Why not? Develop a construction that will work in this case.
8. Let $\triangle ABC$ be a right triangle with $\widehat{C}$ the right angle. How could you choose points X on $\overline{BC}$, Y on $\overline{AC}$ and Z on $\overline{AB}$ to minimize the sum $XY + YZ + XZ$?
9. Let $\triangle ABC$ have a right angle at $\widehat{C}$. prove that the median $\overline{CC'}$ is the Euler line. Is there any other type of triangle in which $\overline{CC'}$ will be the Euler line?