EXAM 1 - MATH 351

Thursday, October 16, 2003
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Read each problem very carefully before starting to solve it. Each question is worth 4 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Show that $3^{2n} - 1$ is exactly divisible by 8 for $n \geq 1$.
   (b) Prove that $n(n-1)2^{n-2} = \sum_{k=1}^{n-1} k(n-k)\binom{n}{k}$.

2. (a) Give a *formal* definition of the terms **subgraph**, **induced subgraph** and **spanning subgraph**. Then explain how many induced and how many spanning subgraphs a labelled graph G has.
   (b) Find a 3-regular graph M such that the graph H below is an induced subgraph of M. Prove that it is impossible to find such an M that has just one vertex more than H has.

3. (a) Define *formally* the notion of **isomorphism** between graphs and list at least 4 isomorphism invariants.
   (b) Draw two different 3-regular graphs on six vertices. Prove that they are not isomorphic.

4. (a) Define *formally* the operations of **join** and of **cartesian product** of graphs. What familiar graph is $K_m + K_n$ isomorphic to? Explain informally.
   (b) Show that, given a positive integer n, there exists a self-complementary graph G with $|V(G)| = 4n$.

5. (a) Give the *formal* definitions of a **tree**, **spanning tree** and **minimum spanning tree**.
   (b) Use Kruskal's algorithm to find the minimum spanning tree of the following weighted graph. Depict all iterations of the algorithm *clearly*!!