EXAM 2 - MATH 351

Thursday, November 20, 2003
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Read each problem very carefully before starting to solve it. Each question is worth 4 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) i. Define the notions of a **bipartite** and of a **semiregular bipartite** graph.
   ii. Let G be a semiregular bipartite graph with parts X and Y, such that $|X| = m$ and $|Y| = n$, $\deg(v) = r$ if $v \in X$, and $\deg(u) = s$ if $u \in Y$. Show that $mr = ns$. Then show that $r \leq n$ and $s \leq m$.

   (b) Suppose that G is bipartite with parts A and B and consider the two copies of G in $G \times K_2$. Call them G_1 and G_2 and their related bipartition sets A_1, B_1 and A_2, B_2, respectively. Show that $G \times K_2$ is bipartite with parts $A_1 \cup B_2$ and $A_2 \cup B_1$.

2. (a) i. Define the notions of **matching**, **maximum matching**, M-**alternating path** and M-**augmenting path**.
   ii. State Berge's Theorem and prove its easiest direction.

   (b) i. Define the notion of a **perfect matching**.
   ii. Show that if a bipartite graph has a perfect matching then it must be equitable. Give a counterexample for the converse of this implication.

3. (a) i. State Hall's Matching Theorem.
   ii. Prove that if a bipartite graph is regular, then it has a perfect matching.

   (b) i. Define an **SDR**.
   ii. Use Hall's Matching Theorem to show that a collection $\{A_1, A_2, \ldots, A_m\}$ of subsets of a set Y has an SDR if and only if $|\bigcup_{i \in S} A_i| \geq |S|$, for all $S \subseteq \{1, 2, \ldots, m\}$.

4. (a) i. Define the notions of **eccentricity**, **radius, diameter**, **center** and **periphery**.
   ii. Prove that if u and v are adjacent vertices in a connected graph, then $|e(u) - e(v)| \leq 1$.

   (b) i. Define the notion of **centroid**.
   ii. Give an example of a tree, such that every vertex in its center is at a distance at least 3 from every vertex in its centroid.

5. (a) i. Define the notions of **vertex connectivity** and **edge connectivity**.
   ii. Show that, given a connected graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

   (b) i. State Menger's Theorem and use the graph below to illustrate the statement.
   ii. Prove Chein's Theorem: A graph G is 2-connected if and only if, for every triple (x, y, z) of distinct vertices, G has an x, z-path through y.