HOMEWORK 2 - MATH 351

DUE DATE: When Chapter 2 has been covered!
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Explain why it is impossible for the list of degrees of the vertices of a graph to be
   (a) $5, 4, 2, 2, 2, 2, 1, 1$ (b) $10, 6, 3, 2, 2, 1, 1, 1$
2. Draw three different 2-regular graphs.
3. Draw a 3-regular graph on six vertices.
4. Explain why there is no 3-regular graph on seven vertices.
5. Describe all 2-regular graphs (connected and disconnected).
6. Find a formula in terms of m and n for the number of edges in the complete bipartite graph $K_{m,n}$.
7. Draw a 3-regular disconnected graph on eight vertices.
8. (a) determine which of the graphs below are subgraphs of graph G. Explain.
   (b) Which of the graphs below are induced subgraphs of graph G? Explain.
9. Why can there be no $a - f$ path of length six in graph G below?
10. Consider the disconnected graph on n vertices consisting of two components: K_{n-1} and K_1. How many edges does it have? Show that this is the maximum number of edges a disconnected graph on n vertices can have.
11. Three nonisomorphic graphs have degree sequences $3, 2, 2, 1, 1, 1$. Construct them.
12. Three nonisomorphic graphs have degree sequence $5, 3, 2, 2, 1, 1, 1, 1$. Construct them.

13. There are three nonisomorphic graphs besides P_7 that have degree sequence $2, 2, 2, 2, 2, 1, 1$. Draw them.

14. Show that the sequence $n - 1, 3, 3, 3, \ldots, 3$ of length $n \geq 4$ is graphical.

15. Given a graph G with degree sequence $d_1, d_2, d_3, \ldots, d_k, d_{k+1}, \ldots, d_n$ show that there exists a graph H with degree sequence (out of order perhaps) $k, d_1 + 1, d_2 + 1, d_3 + 1, \ldots, d_k + 1, d_{k+1}, \ldots, d_n$ by showing how to construct H from G.

16. (a) Prove that C_4 and $K_{2,2}$ are isomorphic.
(b) Prove that $K_{1,2}$ and P_3 are isomorphic.

17. Draw two nonisomorphic disconnected subgraphs of C_5 that have four vertices.

18. Explain why the graphs G and H below are *not* isomorphic by finding some characteristic that distinguishes them from one another.

19. Draw the wheel $W_{1,6}$.

20. ow many edges does $W_{1,n}$ have?

21. Prove that Q_3 is isomorphic to the mesh $M(2, 2, 2)$.

22. Consider K_3 with vertices a, b and c. Now obtain a new graph, A, by adding two more vertices r and s and edges ra and sb. Show that A is self-complementary.

23. Assume that G and H are graphs where $V(G) = \{u_1, \ldots, u_m\}$ and $V(H) = \{v_1, \ldots, v_n\}$. Let (i, j) be a vertex in $G \times H$. Prove that $\deg(i, j) = \deg(u_i) + \deg(v_j)$.

24. Show that $K_{m,n} \cong \overline{K_m} + \overline{K_n}$.

25. Explain why the complement of $G + H$ is disconnected for all pairs of graphs G and H.

26. If $G \cong H$, show that $\overline{G} \cong \overline{H}$.