HOMEWORK 3 - MATH 351

DUE DATE: When Chapter 3 has been covered!
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Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. A **forest** is a graph whose components are trees. There are six nonisomorphic forests that have four vertices. Find them.
2. There are eleven nonisomorphic trees that have seven vertices. Draw them.
3. Suppose that a tree has 50 vertices. How many edges does it have?
4. Show that if a forest F contains c trees and a total of n vertices, then the number of edges in F is $n - c$.
5. Prove that if T_1 and T_2 are trees with n_1 and n_2 vertices, respectively, then the join $T_1 + T_2$ has $n_1 + n_2$ vertices and $(n_1 + 1)(n_2 + 1) - 3$ edges.
6. A rooted tree is called **binary** if each vertex has at most two children. A finite binary tree is called **complete** if each vertex, except each leaf, has exactly two children. How many vertices are there in a complete binary tree of height k? How many leaves are there?
7. If G has n vertices and $n - 1$ edges, must G be a tree? Explain.
8. Find all nonisomorphic spanning trees for the following graphs:
   (a) The wheel $W_{1,5}$ (b) $K_{3,3}$
9. Produce spanning trees of $M(3,3)$ with 2,3,4,5 and 6 end vertices.
10. (a) For the weighted graphs below, list the edges of the spanning tree in the order in which they would be selected if Kruskal's algorithm were used. Then draw the resulting minimum spanning tree.
    (b) List the edges of the spanning tree in the order in which they would be selected if Prim's algorithm were used *beginning at vertex c* in the graph on the left and *beginning at vertex g* in the graph on the right. Then draw the resulting spanning tree.

11. Draw the seven bipartite graphs (both connected and disconnected) that have four vertices.

12. Draw all connected bipartite graphs with six vertices.

13. Suppose that G and H are graphs, at least one of which has an edge. Show that the join $G + H$ is not bipartite.

14. Prove that if a bipartite graph with parts V_1 and V_2 is regular, then $|V_1| = |V_2|$.

15. A graph is **semiregular bipartite** if vertices in part V_1 all have degree s and vertices in part V_2 all have degree t. Prove that if G is semiregular bipartite, then the line graph $L(G)$ is regular of degree $s + t - 2$.

16. Prove that if G_1 and G_2 are bipartite, then so is the Cartesian product $G_1 \times G_2$.

17. If G is semiregular bipartite with n vertices of degree s and m vertices of degree t, determine the number of edges in G.

18. Prove that $G \times K_2$ always has a perfect matching for all graphs G.

19. Given a positive integer n, construct a graph of order n such that a maximum matching has exactly one edge.

20. Let T be a spanning tree of G. Show that a perfect matching for T is also a perfect matching for G Find an example to show that the converse is not true.

21. Find two maximum matchings for each of the following two graphs.

22. (a) Let G be the cycle C_{2n} with vertices labeled $1, 2, 3, \ldots, 2n$. How many different maximum matchings does G have? (b) Let H be the cycle C_{2n+1} with vertices labeled $1, 2, \ldots, 2n + 1$. How many different maximum matchings does H have?

23. Applicant A is qualified for jobs a, b, d, e. applican B is qualified for b, c, e. Applicant C is qualified for b, d, e. Applicant D is qualified for a, c and e and applicant E is qualified for a and b.
    (a) Draw the associated bipartite graph.
    (b) Find a maximum matching to create maximum employment.

24. If possible find a system of distinct representatives for each of the collections of sets:
    (a) $A_1 = \{1, 3, 4, 6\}, A_2 = \{2, 4, 5\}, A_3 = \{1, 2, 6\}, A_4 = \{1, 5, 7\}, A_5 = \{1, 3, 4, 5\}$
    (b) $A_1 = \{4, 5, 6\}, A_2 = \{1, 2, 3, 5\}, A_3 = \{2, 4, 6, 8\}, A_4 = \{1, 2, 8\}, A_5 = \{3, 6, 8\}, A_6 = \{1, 4, 6\}$.