HOMEWORK 4 - MATH 351

DUE DATE: After Chapter 4 has been covered!
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Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Given a vertex v of C_n, find $e(v)$. Show that v has either one or two eccentric vertices, depending on the parity of n.

2. Find the radius and diameter of C_n. Do the same for P_n. Show that the center of P_n consists of one or two adjacent vertices, depending on the parity of n.

3. Let H be a spanning subgraph of a graph G. Given vertices u and v in G, show that their distance from one another in H is at least as big as in G, that is $d_H(u,v) \geq d_G(u,v)$.

4. Show that $C(C_n) = V(C_n)$; that is show that the center of an n-cycle consists of all its vertices. A graph with this property is called **self-centered**. Find another class of self-centered graphs.

5. For the sequential join $G = K_1 + K_1 + \overline{K_2} + K_1 + K_1$, determine $\text{rad}(G), \text{diam}(G), C(G)$ and $P(G)$. Then show that G contains a pair of vertices that are mutually eccentric but not antipodal of one another.

6. Prove that the wheel $W_{1,n}$ has a spanning tree with one center vertex also has a spanning tree with two center vertices.

7. Let G and H be graphs, at least one of which is not complete. Show that $\text{diam}(G+H) = 2$. Why must we stipulate that at least one of G or H is not complete?

8. Find the weight of each vertex in the graph below. Find the centroid.

9. Find all cut vertices and bridges for the graph below:

10. Find three different minimal edge cutsets of size 2 for the graph below:

11. Construct a graph G with $\kappa(G) = 2, \lambda(G) = 3, \delta(G) = 4$.

12. Construct a graph G with $\kappa(G) = 3, \lambda(G) = 3, \delta(G) = 5$.

13. Determine $\kappa(G)$ and $\lambda(G)$ for each of the following graphs:
(a) The octahedron $\overline{K_2} + C_4$ (b) The sequential join $K_2 + K_3 + \overline{K_2} + \overline{K_3}$. (c) The cartesian product $P_4 \times C_3$.

14. Draw the line graph $L(W_{1,4})$. Then find $\kappa(L(W_{1,4}))$ and $\lambda(L(W_{1,4}))$.

15. Prove that every k-connected graph on n vertices has at least $\frac{nk}{2}$ edges.

16. Prove that if G is cubic - that is, 3-regular - then $\kappa(G) = \delta(G)$.

17. Prove that if G is k-connected, then the join $K_1 + G$ is $(k+1)$-connected.