HOMEWORK 5 - MATH 351

DUE DATE: After Chapter 5 has been covered!
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Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Find an eulerian circuit for the graph below. How can you tell in advance that this can be done?

2. Show that if G and H are eulerian, so is $G \times H$.

3. Show that if all the vertices of G and H have odd degree, then $G \times H$ is eulerian.

4. Prove that if G is a connected graph where every vertex has odd degree, then its line graph $L(G)$ is eulerian.

5. For which values of m and n is $K_{m,n}$ eulerian?

6. A knight (the one that looks like a horse) can move on a chessboard in an L-shape, either two spaces up or down and one to the side or two spaces to the side and one space up or down. Determine whether it is possible for the a knight to tour an 8×8 chessboard making legal moves, landing in each square exactly once and ending in the square it started.

7. A museum has a floor plan as displayed in the following figure, where each edge represents a hallway along which paintings are displayed. Vertex e is the location of the entrance, and vertex g is the location of the gift shop through which we exit the museum. Find a route through the museum that begins at the entrance e travels along each hallway exactly once and ends at the gift shop g.

8. Draw a connected graph having five vertices that is
(a) hamiltonian and eulerian
(b) hamiltonian and semi-eulerian
(c) eulerian and semi-hamiltonian

(d) eulerian but not even semi-hamiltonian
(e) hamiltonian but not even semi-eulerian
(f) neither traceable nor traversable
(g) semi-hamiltonian and semi-eulerian

9. Graphs M and N below are hamiltonian. Find a hamiltonian cycle in M and N.

10. Prove that if G has a cut vertex, then G is not hamiltonian.

11. how that the ladder $M(n, 2)$, with $n \geq 3$, is not hamilton laceable.

12. To see that $\frac{n}{2}$ is the "best" result for using $\delta(G)$ to prove Hamiltonicity, find a graph G of order 7 for which $\delta(G) = \frac{n-1}{2}$, such that G is not hamiltonian.

13. Prove that a bipartite graph G in which each part has order n, and G has at least $n^2 - n + 2$ edges, must be hamiltonian.

14. Solve the traveling salesman problem and the Chinese postman problem for the graphs below:

15. Find a hamiltonian path in the Petersen graph.

16. Prove that it is not possible to remove edges 9but no vertices) from the Petersen graph so that the resulting graph becomes eulerian.

17. Consider the floor plan given in the figure below. Explain why it is impossible to enter through the front door, travel through each internal doorway exactly once and exit through the rear door.