HOMEWORK 6 - MATH 351

DUE DATE: After Chapter 6 has been covered!
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Consider the graph below. Explain why $\chi(G) \geq 3$. Then find a 3-coloring of G, thereby proving that $\chi(G) = 3$.

2. Prove that $\chi(G + sw) = 4$ for the graph G below.

3. Let H be a subgraph of G. Show that $\chi(G) \geq \chi(H)$.

4. Find β for C_n.

5. Show that $\chi(G \times K_2) = \chi(G)$.

6. Show that K_n is n-critical.

7. Show that $W_{1,4}$ is uniquely colorable while $W_{1,5}$ is not.

8. Produce an edge coloring, with two colors, for K_6 containing exactly two monochromatic triangles.

9. Find a proper edge coloring of $K_{4,5}$ using five colors.

10. Prove that the edge chromatic number of the Petersen graph is four.

11. Explain why it is true that for all graphs G, $\chi_1(G) = \chi(L(G))$.

12. Prove that if G is k-regular of odd order, then $\chi_1(G) = k + 1$.

13. Determine the edge chromatic number for the graphs below.

14. Determine the chromatic number of the graph below. then find an edge e not in G whose addition to G would increase the chromatic number. Then find a missing edge whose addition to G would not alter the chromatic number.

15. Show that the graph below is uniquely 3-colorable. then find an edge xy such that $H - xy$ is not uniquely 3-colorable.

16. Suppose that the chemical lab in Example 6.6 in your textbook decides that they will no longer need or store chemicals eh, j and m.
    (a) Draw the new graph of chemical interactions.
    (b) Is the resulting graph uniquely k-colorable?
    (c) Determine an arrangement of the remaining chemicals in as few storage cabinets as possible.