HOMEWORK 7 - MATH 152

DUE DATE: Monday, November 22
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each homework problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find a closed form for the n-th partial sum and determine whether the series converges by calculating its limit.

   (a) $2 + \frac{2}{5} + \frac{2}{5^2} + \cdots + \frac{2}{5^{k-1}} + \cdots$

   (b) $\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} \cdots + \frac{1}{(k+1)(k+2)} + \cdots$

2. Determine whether the series converges and if so find its sum:

   (a) $\sum_{k=1}^{\infty}(\frac{1}{2^k} - \frac{1}{2^{k+1}})$

   (b) $\sum_{k=2}^{\infty}\frac{1}{k^2-1}$

   (c) $\sum_{k=1}^{\infty} 5^{3k}7^{1-k}$

3. A ball is dropped from a height of 10 meters. Each time it strikes the ground it bounces vertically to a height that is $\frac{3}{4}$ of the preceding height. Find the total distance the ball will travel if if is assumed to bounce infinitely often.

4. (a) Show that $\sum_{k=1}^{\infty}(\frac{1}{k} - \frac{1}{k+2}) = \frac{3}{2}$

   (b) Show that $\frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \cdots = \frac{3}{4}$

5. Use p-series to determine whether the series converges:

$$(a)\ \sum_{k=1}^{\infty} k^{-4/3}, \quad (b)\ \sum_{k=1}^{\infty}\frac{1}{\sqrt[4]{k}}, \quad (c)\ \sum_{k=1}^{\infty}\frac{1}{\sqrt[3]{k^5}}, \quad (d)\ \sum_{k=1}^{\infty}\frac{1}{k^{\pi}}$$

6. Apply the divergence test and state what it tells you about the series.

$$(a)\ \sum_{k=1}^{\infty}\frac{k}{e^k}, \quad (b)\ \sum_{k=1}^{\infty}\ln k, \quad (c)\ \sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}, \quad (d)\ \sum_{k=1}^{\infty}\frac{\sqrt{k}}{\sqrt{k}+3}$$

7. Confirm that the integral test is applicable and use it to determine whether the series converges.

$$(a)\ \sum_{k=1}^{\infty}\frac{k}{1+k^2}, \quad (b)\ \sum_{k=1}^{\infty}\frac{1}{(4+2k)^{3/2}}$$

8. Make a guess about the convergence or divergence of the series and confirm your guess using the comparison test.

$$(a)\ \sum_{k=2}^{\infty}\frac{k+1}{k^2-k}, \quad (b)\ \sum_{k=1}^{\infty}\frac{2}{k^4+k}, \quad (c)\ \sum_{k=1}^{\infty}\frac{5\sin^2 k}{k!}, \quad (d)\ \sum_{k=1}^{\infty}\frac{\ln k}{k}$$