PRACTICE EXAM 4 - MATH 152

DATE: Friday, December 3
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.
GOOD LUCK!!

1. Determine whether the following sequences converge and if so find the limits (**rigorous explanations required in all problems**):

$$(a)\ \{\frac{\ln n}{n}\}_{n=1}^{\infty} \qquad (b)\ \{\cos\frac{\pi n}{2}\}_{n=1}^{\infty} \qquad (c)\ \{\sqrt{n^2+3n}-n\}_{n=1}^{\infty}$$

2. Use any method to show that the sequence at hand is strictly increasing or strictly decreasing:

$$(a)\ \{n-2^n\}_{n=1}^{\infty} \qquad (b)\ \{\frac{5^n}{2^{(n^2)}}\}_{n=1}^{\infty} \qquad (c)\ \{\frac{1}{n+\ln n}\}_{n=1}^{\infty}$$

3. Let $\{a_n\}$ be the sequence defined recursively by $a_1 = 1$ and $a_{n+1} = \frac{1}{2}(a_n + \frac{3}{a_n})$ for $n \geq 1$.
   (a) Show that $a_n \geq \sqrt{3}$ for $n \geq 2$ by finding the minimum value of $\frac{1}{2}(x+\frac{3}{x})$ for $x > 0$.
   (b) Show that $\{a_n\}$ is eventually decreasing by examining either $a_{n+1} - a_n$ or $\frac{a_{n+1}}{a_n}$ and using part (a).
   (c) Show that $\{a_n\}$ converges and find its limit.

4. Determine if the series converges and if so find its sum:

$$(a)\ \sum_{k=1}^{\infty}(-1)^{k-1}\frac{7}{6^{k-1}} \qquad (b)\ \sum_{k=1}^{\infty}\frac{1}{9k^2+3k-2} \qquad (c)\ \sum_{k=5}^{\infty}(\frac{e}{\pi})^{k-1}$$

5. Determine whether the given series converges:

$$(a)\ \sum_{k=1}^{\infty} ke^{-k^2} \qquad (b)\ \sum_{k=1}^{\infty} k^2\sin^2(\frac{1}{k}) \qquad (c)\ \sum_{k=1}^{\infty}\frac{k!}{k^3}$$

6. Find the radius of convergence and the interval of convergence of the following series:

$$(a)\ \sum_{k=1}^{\infty}\frac{5^k}{k^2}x^k \qquad (b)\ \sum_{k=0}^{\infty}\frac{3^k}{k!}x^k$$