

EXAM 1 - MATH 112

DATE: Thursday, September 22

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Read each problem very carefully before starting to solve it. Each question is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the following limits:

(a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{2 - x}$

(b) $\lim_{x \rightarrow 5} \frac{x - 5}{\sqrt{x} - \sqrt{5}}$

(c) $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 10x + 21}$

2. (a) Give the *formal* (not geometric) definition of a function $y = f(x)$ being continuous at $x = a$. State clearly how this formal definition differs from the *formal* definition of the existence of $\lim_{x \rightarrow a} f(x)$.

(b) Consider the function $f(x) = \begin{cases} 2x + b, & \text{if } x < 2 \\ 4, & \text{if } x = 2 \\ -x^2 + 10, & \text{if } x > 2 \end{cases}$

i. Find b so that $\lim_{x \rightarrow 2} f(x)$ exists.

ii. Find b so that $f(x)$ be continuous at $x = 2$.

3. Use the limit definition of the derivative to compute the derivative of the function $f(x) = \sqrt{x - 3}$ at the point $x = 19$. What is the slope of the tangent line to $y = \sqrt{x - 3}$ at $x = 19$?

4. Use the rules for derivatives to compute the derivatives of the following functions:

(a) $f(x) = (2x + 1)(11x - 3)^4$

(b) $g(x) = \left(\frac{8x+3}{5-x}\right)^7$

5. The demand function for a product is $p = \frac{50}{\sqrt{x}}$, for $1 \leq x \leq 8000$, where, as usual, p is the price per item when x items are sold, and the cost function is $C = 0.5x + 500$, for $0 \leq x \leq 8000$. Find the *marginal profit* when 1600 items are sold.

6. Consider the function $f(x) = \frac{x^2}{x-1}$.

(a) Find the equation of the tangent line to $y = f(x)$ at $x = 3$.

(b) Find all x 's, if any, where the tangent line to $y = f(x)$ is horizontal.