

## EXAM 3 - MATH 112

DATE: Tuesday, November 1

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Read each problem very carefully before starting to solve it. Each question is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

**GOOD LUCK!!**

- Use the values at  $x = \frac{1}{2}$ ,  $x = 1$  and  $x = 2$  to roughly sketch the graph of the function  $f(x) = \log_2 x$ .
  - Use graphing techniques to obtain the graph of  $g(x) = \log_2(x - 1) + 2$  from the graph of  $f$  of the previous part.
- Find the derivatives of
  - $f(x) = \frac{e^{-x^2}}{x+2}$
  - $g(x) = x^2 e^{-x}$
- Use implicit differentiation to find the equation of the tangent line to the graph of  $x^2 y^2 + y e^x = 2e^2$  at the point  $(1, e)$ .
- Find the following derivatives:
  - $f(x) = \frac{\ln(x^2-2)}{x+1}$
  - $g(x) = \ln\left(\sqrt[3]{\frac{x(x+1)}{x^2-4}}\right)$
- Solve the following equations:
  - $\frac{1}{8}2^{x-3} = 4^{5x+1} \cdot 16$
  - $\log_6(x - 5) + \log_6(x - 6) = 1$
- Consider the function  $f(x) = x e^{-x}$ . Do the following step-by-step, as asked.
  - Find the domain of  $f$ .
  - Find the  $x$  and the  $y$ -intercepts of  $f$ .
  - Find  $f'(x)$  and  $f''(x)$ .
  - Form the sign table to determine the monotonicity, the concavity, the relative extrema and the inflection points of  $f$ .
  - Roughly sketch the graph of  $f$ .

**AID:** It is given that  $\lim_{x \rightarrow +\infty} f(x) = 0$ ,  $\frac{1}{e} \cong 0.37$  and  $\frac{2}{e^2} \cong 0.27$ .