PRACTICE EXAM 7 - MATH 140

DATE: Friday, December 15

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. An aircraft is spotted by two observers who are 1000 feet apart. As the airplane passes over the line joining them, each observer takes a sighting of the angle of elevation to the plane. The first observer's angle is 47° and the second observer's angle is 35°.
   (a) Make a sketch of the geometry of the problem indicating the position of each observer and denoting by h the height of the airplane, by a the distance of the airplane from the first observer and by b the distance of the airplane from the second observer. (1 point)
   (b) Provide an expression for computing a involving only known quantities. (2 points)
   (c) Use the expression obtained in part (b) to compute the height h of the airplane. (2 points)

2. Use your Half-Angle Formulas and the Law of Cosines to show that in any triangle the following formula holds:
$$\cos\left(\frac{\gamma}{2}\right) = \sqrt{\frac{(a+b+c)(a+b-c)}{4ab}}. \quad \text{(5points)}$$

3. Consider the polar equation $r = -3\sin\theta$.
   (a) Convert the given equation to cartesian coordinates. (2 points)
   (b) Manipulate the equation that you obtained in the previous part so that you may be able to identify its graph. (2 points)
   (c) Sketch its graph on the plane. (1 point)

4. Suppose that $z = -3 + 3\sqrt{3}i$ and $w = -4i$.
   (a) Write z in polar form. (1 point)
   (b) Write w in polar form. (1 point)
   (c) Multiply zw and leave your answer in polar form. (1 point)
   (d) Divide $\frac{z}{w}$ and leave your answer in polar form. (1 point)
   (e) Find z^6 and write it in standard form. (1 point)

5. Find the six complex 6-th roots of $z = -i$. Regardless of the method that you use to identify then, please, present your work and your answers **clearly**! (5 points)