EXAM 1 - MATH 151

DATE: Friday, September 22

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Consider the function
$$f(x)=\begin{cases} 2x+2, & \text{if } x<0\\ \cos x, & \text{if } 0\le x<\frac{\pi}{2}\\ 1, & \text{if } x=\frac{\pi}{2}\\ \sqrt{x-\frac{\pi}{2}}, & \text{if } x>\frac{\pi}{2}\end{cases}$$
   (a) Sketch carefully the graph of f. (2 points)
   (b) Find $\lim_{x\to 0^-} f(x), \lim_{x\to 0^+} f(x), \lim_{x\to \frac{\pi}{2}^-} f(x), \lim_{x\to \frac{\pi}{2}^+} f(x)$. (2 points)
   (c) Use the previous part to find the limits $\lim_{x\to 0} f(x)$ and $\lim_{x\to\frac{\pi}{2}} f(x)$. (1 point)

2. Suppose that $f(x)=\frac{1}{x+3}$ and $g(x)=\sqrt{2-x}$.
   (a) Find the domains $\mathrm{Dom}(f)$ and $\mathrm{Dom}(g)$. (1 point)
   (b) Find a formula for $(g\circ f)(x)$ and simplify. (2 points)
   (c) Find the domain of $g\circ f$. (2 points)

3. (a) Evaluate the following limits without using any substitutivity properties justifying each step: $\lim_{x\to 5}\frac{x^3-4x^2+1}{7x-9}$ and $\lim_{x\to 1}\sqrt[3]{\frac{x+1}{4x+12}}$. (3 points)
   (b) Suppose that $15x-35\le g(x)\le 2x^2+3x-17$ for all x. Find the $\lim_{x\to 3} g(x)$. Justify your answer. (2 points)

4. Find the following limits:
   (a) $\lim_{x\to 2}\frac{x-2}{x^3-8}$ (2 points)
   (b) $\lim_{h\to 0}\frac{\sqrt{3h+25}-5}{h}$ (2 points)
   (c) $\lim_{t\to 0}(\frac{1}{t}-\frac{1}{t^2-t})$ (1 point)

5. Find the following trigonometric limits:
   (a) $\lim_{\theta\to 0}\frac{\sin 5\theta}{2\theta}$ (1 point)
   (b) $\lim_{t\to 0}\frac{\sin^2 3t}{2t^2}$ (2 points)
   (c) $\lim_{\theta\to 0}\frac{\sin\theta}{\theta+\tan\theta}$ (2 points)

6. (a) Look again at the graph of $f(x)$ that you sketched in the first problem. Then fill-in the following table with "Y" for yes and "N" for no's.

	f continuous from left at	f continuous from right at	f continuous at
$x=0$			
$x=\frac{\pi}{2}$			

   (b) Find the value of a so that the function $g(x)=\begin{cases}\frac{x^2+x-2}{x^2+4x-5}, & \text{if } x\ne 1\\ a, & \text{if } x=1\end{cases}$ be continuous at $x=1$.