

# EXAM 4 - MATH 151

DATE: Friday, December 1

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

**GOOD LUCK!!**

1. Find the absolute minima and maxima of the following functions in the indicated interval. Justify your answers.
  - (a)  $f(x) = (x^2 + x)^{2/3}$ ,  $[-2, 3]$  (3 points)
  - (b)  $f(x) = x^4 + 4x$ ,  $(-\infty, +\infty)$  (2 points)
2. Form the sign table and indicate the positions of the relative extrema and the inflection points for the following functions. (You do not have to graph them).
  - (a)  $f(x) = x^4 - 12x^3$  (2 points)
  - (b)  $f(x) = (xe^x)^2$  (3 points)
3. Find the domain, the  $x$ - and  $y$ -intercepts, the asymptotes, (1 point) form the sign table (2 points) and roughly sketch the graph (2 points) of the function  $f(x) = e^{-x^2/2}$ .
4. Find the domain, the  $x$ - and  $y$ -intercepts, the asymptotes, (1 point) form the sign table (2 points) and roughly sketch the graph (2 points) of the function  $f(x) = \ln \sqrt{x^2 + 4}$ .
5. A closed rectangular container with a square base is to have a volume of 2250 square inches. The material for the top and bottom of the container will cost \$2 per square inch and the material for the sides will cost \$3 per square inch. Find the dimensions of the container of least cost. (5 points)
6.
  - (a) State clearly Rolle's Theorem. (0.5 points)
  - (b) Verify that the hypotheses of Rolle's Theorem are satisfied by  $f(x) = \frac{1}{2}x - \sqrt{x}$  on  $[0, 4]$  and, then, find all values of  $c$  in that interval that satisfy the conclusions of the theorem. (2 points)
  - (c) State clearly the Mean Value Theorem. (0.5 points)
  - (d) Verify that the hypotheses of the Mean Value Theorem are satisfied by  $f(x) = x - \frac{1}{x}$  on  $[3, 4]$  and, then, find all values of  $c$  in that interval that satisfy the conclusions of the theorem. (2 points)