HOMEWORK 2 - MATH 151
DUE DATE: Monday, October 1
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Plot the graph of the piece-wise defined function $f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ 2x^2, & \text{if } 0 \le x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$ Is f continuous at 0? Is it continuous at 1? How about continuous from the left and continuous from the right at these two points?

2. Which of the following two functions has a removable discontinuity at a? Explain.
   (a) $f(x) = \frac{x-7}{|x-7|}, \quad a = 7$
   (b) $f(x) = \frac{3-\sqrt{x}}{9-x}, \quad a = 9$

3. Use Theorems 4,5,6 and/or 8 of Section 1.5 to justify **in detail** why the functions $f(x) = \frac{\sin x}{x+1}$ and $g(x) = x^2 + \sqrt{2x-1}$ are continuous at every number in their domain.

4. Find the constant c that makes $f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \ge 4 \end{cases}$ continuous on $(-\infty, +\infty)$.

5. Use continuity to evaluate the limits:
$$\lim_{x \to 4} \frac{5+\sqrt{x}}{\sqrt{5+x}}, \qquad \lim_{x \to \pi} \sin(x + \sin x).$$

6. Use the Intermediate Value Theorem to show that the equation $\sqrt[3]{x} = 1 - x$ has a root in $(0,1)$.

7. Sketch an example of a function f that satisfies the following conditions: $\lim_{x \to -2} f(x) = +\infty$, $\lim_{x \to -\infty} f(x) = 4$ and $\lim_{x \to +\infty} f(x) = -2$.

8. Find the following limits: $\lim_{x \to 1} \frac{x-2}{(1-x)^2}$, $\lim_{x \to \frac{\pi}{2}^+} \sec x$.

9. Find the limits $\lim_{x \to +\infty} (x - \sqrt{x})$, $\lim_{x \to +\infty} (x^2 - x^4)$, $\lim_{x \to +\infty} \frac{4x+7}{\sqrt{9x^2+2}}$.

10. Find a formula for a function that has vertical asymptotes $x = -2$ and $x = 5$ and horizontal asymptote $y = 2$.