HOMEWORK 7 - MATH 151
DUE DATE: Monday, November 19
INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Use L'Hospital's rule, where appropriate, to find the limit.
   (a) $\lim_{x\to(\pi/2)^+} \frac{\cos x}{1-\sin x}$
   (b) $\lim_{x\to\infty} \frac{\ln\ln x}{x}$
   (c) $\lim_{x\to-\infty} x^2e^x$
   (d) $\lim_{x\to\infty} (x - \ln x)$
   (e) $\lim_{x\to 0^+} (\tan 2x)^x$
2. Find the critical numbers of the following functions:
   (a) $f(x) = x^{4/5}(x-4)^2$
   (b) $f(x) = x\ln x$
   (c) $f(x) = xe^{2x}$
3. Find the absolute maximum and the absolute minimum values of f on the given interval:
   (a) $f(x) = x\sqrt{4-x^2}$ in $[-1, 2]$
   (b) $f(x) = x - \ln x$ in $[\frac{1}{2}, 2]$
4. Prove that the function $f(x) = x^{101} + x^{51} + x + 1$ has neither a local maximum nor a local minimum.
5. Verify that the function $f(x) = x^3 - 3x^2 + 2x + 5$ satisfies the three hypotheses of Rolle's Theorem on the interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.
6. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem:
   (a) $f(x) = x^3 + x - 1$ on $[0, 2]$
   (b) $f(x) = \frac{x}{x+2}$ on $[1, 4]$.
7. Show that the equation $1 + 2x + x^3 + 4x^5 = 0$ has exactly one real root.
8. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?
9. Use Theorem 5 on page 209 to prove the identity $2\sin^{-1} x = \cos^{-1}(1-2x^2), x \geq 0$.
10. Find the domains, the intercepts, the asymptotes, form the sign tables and then roughly sketch the graphs of the following functions:

$$(a)\ f(x) = x^4 - 4x - 1 \quad (b)\ f(x) = x^2e^x \quad (c)\ f(x) = x\ln x$$