EXAM 2: SOLUTIONS - MATH 111
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Problem 1 *Find the vertex, the opening direction, the x- and y-intercepts and sketch the graph of* $f(x) = \frac{1}{2}x^2 - 3x + \frac{5}{2}$.

Solution:

The vertex has x-coordinate

$$x = -\frac{b}{2a} = -\frac{-3}{2 \cdot \frac{1}{2}} = 3$$

and y-coordinate $f(1) = \frac{1}{2}3^2 - 3 \cdot 3 + \frac{5}{2} = -2$. Hence, it is the point $(3, -2)$.

The parabola opens up, since $a = \frac{1}{2} > 0$.

The y-intercept is found by setting $x = 0$. We have then $y = \frac{5}{2}$. Thus $(0, \frac{5}{2})$ is the y-intercept. The x-intercepts are found by setting $y = 0$ and solving $\frac{1}{2}x^2 - 3x + \frac{5}{2} = 0$. We have $x^2 - 6x + 5 = 0$, i.e., $(x-5)(x-1) = 0$, whence $x = 1$ or $x = 5$. Thus, the x-intercepts are the points $(1, 0)$ and $(5, 0)$.

The graph follows

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Problem 2 *Find the equation of the parabola that has vertex* $V = (-1, 3)$ *and goes through the point* $(0, 1)$.

Solution:

Since the vertex is at $V = (-1, 3)$, we have equation

$$f(x) = a(x+1)^2 + 3.$$

But the parabola goes through $(0, 1)$, whence

$$1 = a(0+1)^2 + 3, \quad \text{i.e.,} \quad 1 = a + 3,$$

which yields $a = -2$. Thus, the equation of the parabola is

$$f(x) = -2(x+1)^2 + 3.$$

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Problem 3 *The supply and the demand of a specific item are modelled by* $p = q^2 + q + 5$ *and* $p = -q^2 + 5q + 35$*, respectively, where* p *denotes price and* q *number of items. Find the equilibrium price and the equilibrium supply.*

Solution:
To find the equilibrium supply set the two price functions equal:

$$q^2 + q + 5 = -q^2 + 5q + 35.$$

This gives $2q^2 - 4q - 30 = 0$. By dividing by 2 both sides, we get $q^2 - 2q - 15 = 0$. This quadratic factors as $(q-5)(q+3) = 0$. Hence $q = -3$ or $q = 5$. Since supply has to be positive, we get $q = 5$. The equilibrium price is then $p = 5^2 + 5 + 5 = 35$. ■

Problem 4 *Find the vertical and the horizontal asymptotes and the* x*- and* y*-intercepts of the function* $f(x) = \frac{-x+1}{x-4}$ *and roughly sketch its graph.*

Solution:
The vertical asymptote is $x = 4$. The horizontal asymptote is given by $y = -1$. The y-intercept is obtained by setting $x = 0$. We get $y = -\frac{1}{4}$. Thus, it is the point $(0, -\frac{1}{4})$. The x-intercept is found by setting $y = 0$. This gives $x = 1$. The graph follows

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Problem 5 *Solve the exponential equation* $9^{x^2-8} = 3^{-14x}$.

Solution:
We have by transforming both sides to base 3: $(3^2)^{x^2-8} = 3^{-14x}$. Hence $2(x^2-8) = -14x$, whence $x^2 - 8 = -7x$ and, therefore, $x^2 + 7x - 8 = 0$. This factors as $(x+8)(x-1) = 0$. Therefore, we obtain the two solutions $x = -8$ or $x = 1$. ■

Problem 6 *Solve the logarithmic equation* $\log_3 x - \log_3 (x-5) = 2$.

Solution:
We have $\log_3 x - \log_3 (x-5) = 2$ implies $\log_3 \frac{x}{x-5} = 2$, whence $\frac{x}{x-5} = 3^2$, i.e., $\frac{x}{x-5} = 9$. Multiply both sides by $x - 5$ to get $x = 9(x-5)$. This gives $x = 9x - 45$, whence $8x = 45$, i.e., $x = \frac{45}{8}$. Check to see that this is an accepted solution. ■