EXAM 1: SOLUTIONS - MATH 111
INSTRUCTOR: George Voutsadakis

Problem 1 *Find the equation of the line that is parallel to* $3x+5y=21$ *and passes through the point* $(3,-4)$.

Solution:
The slope of the given line may be found by solving its equation for y: We have: $5y=-3x+21$, whence $y=-\frac{3}{5}x+\frac{21}{5}$. Hence $m=-\frac{3}{5}$. The unknown line has is parallel to the given one, whence its slope is also m. Since, it goes through the point $(3,-4)$, its equation is given by the point-slope form:

$$y-(-4)=-\frac{3}{5}(x-3), \quad \text{i.e.,} \quad y+4=-\frac{3}{5}x+\frac{9}{5},$$

or $y=-\frac{3}{5}x-\frac{11}{5}$. ■

Problem 2 *Find the equation of the line that is perpendicular to the line* $y=5x+2003$ *and passes through the point* $(-10,8)$.

Solution:
The slope of the given line is $m=5$. Hence, since the wanted line is perpendicular to that, its slope must be $-\frac{1}{5}$. This, together with the fact that it goes through the point $(-10,8)$, yield the equation

$$y-8=-\frac{1}{5}(x-(-10)), \quad \text{i.e.,} \quad y-8=-\frac{1}{5}x-2$$

or $y=-\frac{1}{5}x+6$. ■

Problem 3 *The cost* C *in terms of the number of items* x *produced is given by* $C(x)=3x+120$ *and the revenue by* $R(x)=7x$. *Find the range of values of* x *for which the company will at least break even and the revenue, when the company breaks even.*

Solution:
The company will at least break even when $R(x)\geq C(x)$. Thus we have $7x\geq 3x+120$, which gives $4x\geq 120$, i.e., $x\geq 30$.
The revenue when the company breaks even is given by $R(30)=7\cdot 30=210$. ■

Problem 4 *The demand price* p *of an item in terms of the quantity* q *is given by* $p=-q^2+3600$ *and the supply price* p *in term of the quantity* q *by* $p=50q$. *Determine the equilibrium price and the equilibrium supply.*

Solution:
To find the equilibrium quantity (both supply and demand) we set $-q^2+3600=50q$, whence $q^2+50q-3600=0$, i.e., $(q+90)(q-40)=0$. Therefore $q=-90$ or $q=40$. Since q denotes quantity, it cannot be negative, whence we get $q=40$. Now the equilibrium price is given by $p=50\cdot 40=2000$. ■

Problem 5 *Solve the inequality* $|x-7|-1 \leq 20$.

Solution:

We have $|x-7|-1 \leq 20$ implies $|x-7| \leq 21$, whence $-21 \leq x-7 \leq 21$, and, therefore, $-14 \leq x \leq 28$. ■

Problem 6 *Find the domain of* $f(x) = \sqrt{\frac{-x+1}{x+2}}$.

Solution:

First $x+2 \neq 0$, whence $x \neq -2$. Since $\frac{-x+1}{x+2}$ appears underneath a square root, we must also have $\frac{-x+1}{x+2} \geq 0$. We may now set up the sign table to find $-2 < x \leq 1$. ■