# EXAM 3: SOLUTIONS - MATH 111
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**Problem 1** *Solve the logarithmic equation* $(\log x)^3 = \log(x^4)$.

**Solution:**

We have $(\log x)^3 = \log(x^4)$ implies $(\log x)^3 = 4\log x$, whence $(\log x)^3 - 4\log x = 0$, i.e., $\log x((\log x)^2 - 4) = 0$. Therefore $\log x(\log x - 2)(\log x + 2) = 0$. This yields $\log x = 0$ or $\log x = 2$ or $\log x = -2$. Thus $x = 10^0$ or $x = 10^2$ or $x = 10^{-2}$, i.e., the solutions are $x = 1$ $x = 100$ or $x = \frac{1}{100}$. ■

**Problem 2** *Brian deposits \$1,000 at the end of each quarterly period for 2 years in an account paying 8% compounded quarterly. After this period, he leaves the money alone with no further deposits for an additional 3 years. Find the final amount in the account at the end of the entire 5 year period.*

**Solution:**

For the first 2 years Brian has set up an ordinary annuity paying 8% quarterly, whence its future amount after the 2 year period will be $A_2 = R\frac{(1+i)^n-1}{i} = 1000\frac{(1+0.02)^8-1}{0.02} = 1000\frac{1.02^8-1}{0.02}$. Then the amount $A_2$ is left to accumulate compound interest under the same terms for another 3 years. Thus the future amount after the entire 5 year period is

$$A = A_2(1+0.02)^{12} = 1000\frac{1.02^8-1}{0.02}1.02^{12}.$$

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**Problem 3** *Solve the following system* **by substitution** $\left\{\begin{array}{rcrcr} 2x & - & 5y & = & 16 \\ 7x & - & 3y & = & 27 \end{array}\right\}$

**Solution:**

The first equation gives $x = \frac{5}{2}y + 8$. Substitute into the second equation to get $7(\frac{5}{2}y + 8) - 3y = 27$, whence $\frac{35}{2}y + 56 - 3y = 27$. Hence $\frac{29}{2}y = -29$. Thus yields $y = -2$. Now substituting this value into the first equation we get back $x = \frac{5}{2}(-2) + 8 = 3$. Thus, the given system has the unique solution $(x, y) = (3, -2)$. ■

**Problem 4** *Solve the following system* **by the Gauss-Jordan method**

$$\left\{\begin{array}{rcrcrcr} x & - & y & + & z & = & -4 \\ -2x & + & y & - & z & = & 5 \\ -x & - & 2y & + & z & = & -3 \end{array}\right\}.$$

**Solution:**

We have

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & -4 \\ -2 & 1 & -1 & 5 \\ -1 & -2 & 1 & -3 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & -1 & 1 & -4 \\ 0 & -1 & 1 & -3 \\ 0 & -3 & 2 & -7 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & -1 & 1 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & -3 & 2 & -7 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right].$$

Thus, the given system has the unique solution $(x, y, z) = (-1, 1, -2)$. ■

**Problem 5** *Pretzels cost \$3 per pound, dried fruit \$4 per pound and nuts \$8 per pound. How many pounds of each should be used to produce 140 pounds of trail mix costing \$6 per pound in which there are twice as many pretzels (by weight) as dried fruit?*

**Solution:**

Let $x, y$ and $z$ be the quantities in pounds of pretzels, dried fruit and nuts, respectively, that have to be mixed to produce the 140 pounds of the trail mix. Then we have

$$\left\{\begin{array}{cccc} x & +y & +z & = & 140 \\ 3x & +4y & +8z & = & 840 \\ x & -2y & & = & 0. \end{array}\right\}.$$

The third equation gives $x = 2y$. Thus, the first and the second become $3y + z = 140$ and $10y + 8z = 840$, respectively. The first of these gives $z = 140 - 3y$. The second yields $5y + 4z = 420$, whence $5y + 4(140 - 3y) = 420$, i.e., $5y + 560 - 12y = 420$. Thus $7y = 140$ or $y = 20$. Thus $x = 40$ and $z = 80$. We need to mix 40 pounds of pretzels, 20 pounds of dried fruit and 80 pounds of nuts. ■

**Problem 6** *Solve the matrix equation $2A + X = 3B$, where $A = \begin{bmatrix} -1 & 3 \\ 5 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$.*

**Solution:**

From $2A + X = 3B$ we obtain

$$X = 3B - 2A = 3\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} - 2\begin{bmatrix} -1 & 3 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} -2 & 6 \\ 10 & -4 \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ -10 & -5 \end{bmatrix}.$$

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