EXAM 4: SOLUTIONS - MATH 111
INSTRUCTOR: George Voutsadakis

Problem 1 *Let* $U = \{1, 2, 3, \dots, 100\}$. *In* U *consider the sets* $A = \{61, 62, 63, \dots, 100\}$, $B = \{1, 2, 3, \dots, 45\}$ *and* $C = \{2, 4, 6, \dots, 100\}$. *Find the set* $(C \cup (A \cap B')') \cap C'$. *(Please, give an* **unambiguous description** *of that set.)*

Solution:

$$\begin{aligned}
(C \cup (A \cap B')') \cap C' &= (\{2,4,\dots,100\} \cup (\{61,62,\dots,100\} \cap \{1,2,\dots,45\}')') \cap \{2,4,\dots,100\}' \\
&= (\{2,4,\dots,100\} \cup (\{61,62,\dots,100\} \cap \{46,47,\dots,100\})') \cap \{1,3,\dots,99\} \\
&= (\{2,4,\dots,100\} \cup \{61,62,\dots,100\}') \cap \{1,3,\dots,99\} \\
&= (\{2,4,\dots,100\} \cup \{1,2,\dots,60\}) \cap \{1,3,\dots,99\} \\
&= \{1,2,\dots,60,62,64,\dots,100\} \cap \{1,3,\dots,99\} \\
&= \{1,3,\dots,59\}
\end{aligned}$$

■

Problem 2 *A polling of 100 TV viewers over the coverage of the Iraqi war has revealed that 28 were following CNN, 42 were watching Al Jazeerah and 56 were watching the BBC. Of the 100 viewers, 7 were watching both CNN and Al Jazeerah, 17 both Al Jazeerah and the BBC, whereas 9 were watching both CNN and the BBC. Finally, 2 viewers said that they were following coverage on all three channels. How many viewers were not following the war stories in any of these three major channels?*

Solution:

Let C denote the set of CNN viewers, A those of Al Jazeerah and B BBC's viewers. Then the different areas of the Venn diagram are filled as follows:

Area	Number
$C \cap A \cap B$	2
$C \cap A \cap B'$	5
$C \cap A' \cap B$	7
$C' \cap A \cap B$	15
$C \cap A' \cap B'$	14
$C' \cap A \cap B'$	20
$C' \cap A' \cap B$	32
$C' \cap A' \cap B'$	5

Hence 5 people did not follow the war in any of the three channels. ■

Problem 3 *A survey was conducted to explore the preference of a population concerning red and white wine. 15% of the people surveyed liked only white wine. 60% did not like red wine or did not like white wine, whereas 80% liked at least one of the two kinds of wine. Find the odds for a person selected at random in the surveyed population liking* only *red wine.*

Solution:

We have that $P(R' \cap W) = 0.15, P(R' \cup W') = 0.60$ and $P(R \cup W) = 0.80$. The second one gives $P((R \cap W)') = 0.60$, whence $P(R \cap W) = 0.40$. Therefore the four areas of the corresponding Venn diagram should be filled as follows:

Area	Number
$R \cap W$	0.40
$R \cap W'$	0.25
$R' \cap W$	0.15
$R' \cap W'$	0.20

Hence, the odds for liking only red wine are $\frac{P(R \cap W')}{P((R \cap W')')} = \frac{0.25}{0.75} = \frac{1}{3}$. ■

Problem 4 *Consider the experiment of tossing 2 fair coins and rolling a die. Find the probability of two heads and a 2 appearing given that at least one head and an even number appeared.*

Solution:

$$P(2H\&2| \geq 1H\&Even) = \frac{P(2H\&2)}{P(\geq 1H\&Even)} = \frac{\frac{1}{24}}{\frac{9}{24}} = \frac{1}{9}.$$

■

Problem 5 *A jar contains 5 red, 4 black, 3 white and 8 green balls. Two balls are drawn at random without replacement. Find the probability of*

1. *the first ball being green and the second being black,*
2. *one ball being red and one being white.*

Solution:

$$P(1stG \cap 2ndB) = P(2ndB|1stG)P(1stG) = \frac{4}{19}\frac{8}{20}.$$

$$\begin{aligned} P(1R\&1W) &= P(1stR \cap 2ndW) + P(1stW \cap 2ndR) \\ &= P(2ndW|1stR)P(1stR) + P(2ndR|1stW)P(1stW) \\ &= \tfrac{3}{19}\tfrac{5}{20} + \tfrac{5}{19}\tfrac{3}{20}. \end{aligned}$$

■

Problem 6 *A survey was conducted on smoking in a specific area. In that area 10% of the population surveyed were classified as (active) smokers, 25% as passive smokers and the remaining 65% were non-smokers (neither active nor passive). The survey yielded 30% of the smoking population suffering from lung cancer, whereas the corresponding results for passive smokers and non-smokers were 10% and 5%, respectively. If a person selected at random has lung cancer, what is the probability that she/he had been classified as a passive smoker?*

Solution:

Let S, P, N and L denote the events of a surveyed individual being a smoker, a passive smoker, a non-smoker and a lung cancer patient, respectively. Then

$$P(P|L) = \frac{P(L|P)P(P)}{P(L|S)P(S) + P(L|P)P(P) + P(L|N)P(N)} = \frac{0.1 \cdot 0.25}{0.3 \cdot 0.1 + 0.1 \cdot 0.25 + 0.05 \cdot 0.65}$$

■