HOMEWORK 10: SOLUTIONS - MATH 111
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Problem 1 *In a U.S. state, 20% of the population lives in inner cities, 35% in suburbs and 45% in rural areas. 20% of those living in inner cities receive poor medical care and the corresponding probabilities for those living in the suburbs and in rural areas are 5% and 10%, respectively. Find the probability that a person in the population selected at random receives satisfactory care.*

Solution:

Let F denote the event of receiving satisfactory health care, I the event of living in an inner city, S the event of living in a suburb and R the event of living in a rural area. Then

$$\begin{aligned} P(F) &= P(F \cap I) + P(F \cap S) + P(F \cap R) \\ &= P(F|I)P(I) + P(F|S)P(S) + P(F|R)P(R) \\ &= 0.8 \cdot 0.2 + 0.95 \cdot 0.35 + 0.9 \cdot 0.45 \end{aligned}$$

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Problem 2 *In the country Utopia, the official language is Utopic, whose alphabet has only 16 letters, and the numbering system provides only for 7 digits. In that country, the licence plates of registered vehicles consist of two pairs: the first pair consists of a letter followed by a number and the second pair consists of a number followed by a letter.*

1. *How many Utopic licence plates are possible?*
2. *How many are possible if the beginning number of the second pair is not allowed to be that one number of the seven that represents 0?*

Solution:

Since there are 16 choices for letters and 7 choices for numbers, the total number of possible Utopic license plates is $7^2 16^2$. If, however, the number in the second pair is forbidden from being 0, then the total number of possible plates becomes $6 \cdot 7 \cdot 16^2$. ■

Problem 3 *How many different "words" may formed by using all the letters in the word "TENNESSEE"?*

Solution:

Since there are 9 letters that are divided into 4 groups with 1,2,2 and 4 letters each, the possible number of words in these letters is $\frac{9!}{(2!)^2 4!}$. ■

Problem 4 *The U.S. senate has 53 republican and 47 democratic senators. A committee of 11 members is to be formed consisting of 7 republicans and 4 democratic senators. In how many ways is it possible to form such a committee?*

Solution:

We first choose the democratic, then the republican senators and, finally, we apply the multiplication principle to form the committees of 11: $\binom{47}{7}\binom{53}{7}$. ■

Problem 5 *A bridge hand consists of 13 cards out of a normal deck of 52 cards. Find the probability that a bridge hand contains*

1. *5 face cards.*
2. *5 cards of one suit and 8 of another.*

Solution:

$$P(5 \text{ Face}) = \frac{\binom{12}{5}\binom{40}{8}}{\binom{52}{13}}$$

$$P(5 \text{ of one and } 8 \text{ of another}) = \frac{4 \cdot \binom{13}{5} \cdot 3 \cdot \binom{13}{8}}{\binom{52}{13}}.$$

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Problem 6 *Suppose that a government agency has a board consisting of 8 Caucasian, 3 Hispanic and 4 African American members. A committee of 5 members of this board is to be formed to deal with issues concerning Hispanics. In how many ways can such a committee be formed so that at least one of the Hispanic board members is also a member of the committee?*

Solution:

From the total number of possible committees subtract the number of those that do not contain any Hispanic board members: $\binom{15}{5} - \binom{12}{5}$. ■

Problem 7 *A coin is tossed 7 times. What is the probability of obtaining at least 5 heads? What is the probability of no more than 2 tails?*

Solution:

$$\begin{aligned} P(\geq 5H) &= P(5H) + P(6H) + P(7H) \\ &= \tbinom{7}{5}(\tfrac{1}{2})^7 + \tbinom{7}{6}(\tfrac{1}{2})^7 + \tbinom{7}{7}(\tfrac{1}{2})^7. \end{aligned}$$

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Problem 8 *A certain machine produces a defective item with probability 0.05. What is the probability that out of 100 items manufactured by this machine at least one defective item is produced?*

Solution:

$$\begin{aligned} P(\geq 1D) &= 1 - P(0D) \\ &= 1 - \tbinom{100}{0}0.05^0 0.95^{100} \\ &= 1 - 0.95^{100}. \end{aligned}$$

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