HOMEWORK 4: SOLUTIONS - MATH 111
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Problem 1 *Find the domain of the function* $f(x) = \sqrt{\frac{x-3}{x^2+2x-3}}$.

Solution:

We need to ensure that $x^2 + 2x - 3 \neq 0$ and that $\frac{x-3}{x^2+2x-3} \geq 0$. The first one gives $(x + 3)(x - 1) \neq 0$, whence we must have $x \neq -3$ and $x \neq 1$. The second one gives $\frac{x-3}{(x+3)(x-1)} \geq 0$. we may now construct the sign table for the fraction $\frac{x-3}{(x+3)(x-1)}$ and this will give us that $\frac{x-3}{x^2+2x-3} \geq 0$ if $-3 < x < 1$ or $x \geq 3$. Hence $D(f) = \{x : -3 < x < 1 \text{ or } x \geq 3\}$.

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Problem 2 *Graph the piece-wise linear function*

$$f(x) = \begin{cases} 2x + 1, & \textit{if } x \leq 2 \\ -x + 3, & \textit{if } x > 2 \end{cases}$$

Solution:

we graph $2x - 1$ and keep the part of its graph for $x \leq 2$ and on the same system of coordinates then graph $-x + 3$ and keep the part with $x > 2$. The graph would have been shown below:

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Problem 3 *Consider the function* $g(x) = -x^2 + 7x - 10$. *Its graph is a parabola. Find its vertex and x-intercepts, state whether it opens up or down and make a rough sketch of it.*

Solution:

The vertex is $V = (-\frac{b}{2a}, f(-\frac{b}{2a})) = (-\frac{7}{2(-1)}, f(\frac{7}{2})) = (\frac{7}{2}, \frac{9}{4})$. For the x-intercepts, we set $y = 0$ and find $-x^2 + 7x - 10 = 0$, whence $x^2 - 7x + 10 = 0$, i.e., $(x - 5)(x - 2) = 0$ and we have $x = 2$ or $x = 5$. the graph opens down since $a = -1 < 0$. The sketch would have appeared here:

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Problem 4 *Consider the function $g(x) = x^2 + 4x$. Its graph is a parabola. Find its vertex and x-intercepts, state whether it opens up or down and make a rough sketch of it.*

Solution:

The vertex is $V = (-\frac{b}{2a}, f(-\frac{b}{2a})) = (-\frac{4}{2\cdot 1}, f(-\frac{4}{2})) = (-2, -4)$. For the x-intercepts, we set $y = 0$ and find $x^2 + 4x = 0$, whence $x(x + 4)$, i.e., $x = -4$ or $x = 0$. the graph opens up since $a = 1 > 0$. The sketch would have appeared here:

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Problem 5 *Find the equation of the function whose graph is a parabola with vertex $V = (2, 3)$ passing through $(-1, -1)$.*

Solution:

In the form $f(x) = a(x - h)^2 + k$ we have that the vertex is located at $(h, k) = (2, 3)$. Whence the equation is $f(x) = a(x - 2)^2 + 3$. But the parabola also goes through the point $(-1, -1)$, whence we must have

$$-1 = a(-1 - 2)^2 + 3, \text{ i.e., } -1 = a(-3)^2 + 3$$

which yields $9a+3 = -1$, and therefore $a = -\frac{4}{9}$. Hence the equation is $f(x) = -\frac{4}{9}(x-2)^2+3$. ■

Problem 6 *When the price of a bizz is $p(x) = 100 - 2x$, then x bizz are sold. Find an expression for the revenue $R(x)$ in terms of the number x of bizz. Find the number of bizz that have to be sold to maximize the revenue and the maximum revenue.*

Solution:

We know that the revenue is given by the product of the number of items sold times the price of each item. Thus $R(x) = x(100 - 2x)$ which yields an equation $R(x) = -2x^2 + 100x$ which is quadratic in x, i.e., whose graph is a parabola and it opens down since $a = -1 < 0$. Hence it has a maximum that is attained at its vertex: $V = (-\frac{b}{2a}, R(-\frac{b}{2a})) = (-\frac{100}{2(-2)}, R(\frac{100}{25})) = (25, 1250)$. Thus the maximun revenue is \$1,250 and occurs when 25 bizz are sold. ■

Problem 7 *An object is thrown upward with initial velocity 10 feet per second from an initial height of 11 feet. Then its height after t seconds is given by* $h(t) = -t^2 + 10t + 11$. *Find the maximum height that the object will attain and how long it will take for the object to hit the ground.*

Solution:

The maximum height will be attained at the vertex of the parabola. hence we have $V = (-\frac{b}{2a}, h(-\frac{b}{2a})) = (-\frac{10}{2(-1)}, h(\frac{10}{2})) = (5, h(5)) = (5, 36)$. Thus after $t = 5$ seconds the object will reach its maximum height $h = 36$ feet. The object will hit the ground when $h = 0$. Thus we have $-t^2 + 10t + 11 = 0$, whence $t^2 - 10t - 11 = 0$, i.e., $(t - 11)(t + 1) = 0$ and therefore $t = -1$ or $t = 11$. But time cannot be negative, whence the object will hit the ground after $t = 11$ seconds. ■

Problem 8 *Create the sign table and graph the function* $f(x) = x^3 - 3x^2$.

Solution:

First factor into linear terms to find the zeros of the function: $x^3 - 3x^2 = 0$ implies $x^2(x - 3) = 0$, whence $x = 0$ or $x = 3$. Create now the sign table to find that $f(x) \leq 0$, if $x \leq 3$ and $f(x) \geq 0$ if $x \geq 3$. Now put the zeros on your coordinate system and plot the graph according to the data in the sign table.

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