

HOMWORK 6: SOLUTIONS - MATH 111

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Problem 1 If $\ln x = 3$ and $\ln y = 4$ find $\ln\left(\frac{x^2}{\sqrt{y}}\right)$.

Solution:

We have

$$\begin{aligned}\ln\left(\frac{x^2}{\sqrt{y}}\right) &= \ln(x^2) - \ln\sqrt{y} \\ &= 2\ln x - \ln(y^{\frac{1}{2}}) \\ &= 2\ln x - \frac{1}{2}\ln y \\ &= 2 \cdot 3 - \frac{1}{2} \cdot 4 \\ &= 4.\end{aligned}$$

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Problem 2 Solve the equation $\log_2(x-1) - \log_2(x-5) = 3$.

Solution:

$\log_2(x-1) - \log_2(x-5) = 3$ implies $\log_2\frac{x-1}{x-5} = 3$ which gives $\frac{x-1}{x-5} = 2^3$, i.e., $x-1 = 8(x-5)$. This yields $x-1 = 8x-40$, whence $7x = 39$, and, therefore, $x = \frac{39}{7}$. ■

Problem 3 Solve the equation $\log_{39}(x+1) + \log_{39}(x-9) = 1$.

Solution:

We have $\log_{39}(x+1) + \log_{39}(x-9) = 1$ implies $\log_{39}(x+1)(x-9) = 1$, whence $(x+1)(x-9) = 39^1$, i.e., $x^2 - 8x - 9 = 39$ or $x^2 - 8x - 48 = 0$. This has the factorization $(x-12)(x+4) = 0$, whence $x = -4$ or $x = 12$. However, since the two logarithms $\log_{39}(x+1)$ and $\log_{39}(x-9)$ are not allowed to have negative arguments, $x = -4$ is not an acceptable solution. Only $x = 12$ is acceptable! ■

Problem 4 Solve the equation $\log(x^4) = (\log x)^2$.

Solution:

$\log(x^4) = (\log x)^2$ gives $4\log x = (\log x)^2$, whence $(\log x)^2 - 4\log x = 0$. Hence $\log x(\log x - 4) = 0$. Therefore $\log x = 0$ or $\log x = 4$. Thus the solutions for x are $x = 10^0$ or $x = 10^4$, i.e., $x = 1$ or $x = 10000$. ■

Problem 5 The growth of an outpatient surgery as a percent of total surgeries at hospitals is approximated by $f(x) = -1317 + 304\ln x$, where x represents the number of years since 1900.

(a) What does this function predict for the percent of outpatient surgeries in 2004?

(b) When did outpatient surgeries reach 50%?

Solution:

(a) We have $f(104) = -1317 + 304 \ln 104 = 94.89$.

(b) For this part, we are seeking x such that $f(x) = 50$. We have $-1317 + 304 \ln x = 50$ implies $304 \ln x = 1367$, whence $\ln x = \frac{1367}{304}$, i.e., $x = e^{\frac{1367}{304}} = 89.72$. Thus, the year will be $1900 + 89 = 1989$.

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Problem 6 Find the simple interest on a loan of \$20,000 at 4% made on September 1 and due on November 30.

Solution:

We have $I = Prt = 20000 \cdot 0.04 \cdot \frac{3}{12} = 20000 \cdot 0.01 = 200$.

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Problem 7 A friend of yours decided to go back to college. She decides to buy a small car for \$7,000. She intends to borrow the money from a bank with 10% discount rate. If she plans to repay the loan in 3 years what will be the amount of her loan?

Solution:

We have $P = A(1 - rt)$, whence $A = \frac{P}{1-rt} = \frac{7000}{1-0.1 \cdot 3} = \frac{7000}{0.7} = 10,000$.

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Problem 8 Find the amount of interest earned by a deposit of \$5,000 compounded quarterly at 4% for 5 years.

Solution:

$I = A - P = P(1 + \frac{r}{m})^{mt} - P = 5000(1 + \frac{0.04}{4})^{4 \cdot 5} - 5000 = 5000(1.01)^{20} - 5000 = 1,100.95$.

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