HOMEWORK 7: SOLUTIONS - MATH 111
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Problem 1 *Find the present value of the future amount \$10,000 compounded semiannually at 6% for 5 years.*

Solution:
We have $A = P(1+\frac{r}{m})^{mt}$, i.e., $P = \frac{A}{(1+\frac{r}{m})^{mt}}$, whence $P = \frac{10000}{(1+\frac{0.06}{2})^{2\cdot 5}}$ and, therefore, $P = \frac{10000}{1.03^{10}}$. ■

Problem 2 *Find the sum of the first five terms of the geometric sequence with first term $a = 3$ and common ratio $r = 2$.*

Solution:
$S_5 = a\frac{r^n-1}{r-1} = 3\frac{2^5-1}{2-1} = 3 \cdot 31 = 93.$ ■

Problem 3 *Solve the systems*

$$\left\{\begin{array}{rcrcr} 3x & + & 5y & = & 7 \\ -x & + & 2y & = & 5 \end{array}\right\}, \quad \left\{\begin{array}{rcrcr} -2x & + & 3y & = & 1 \\ 8x & - & 12y & = & -4 \end{array}\right\},$$

by the substitution method.

Solution:

(a) We have $x = 2y - 5$, by solving the second equation for x. Now substituting into the first equation, we get $3(2y-5)+5y = 7$, whence $6y - 15 + 5y = 7$ and, thus, $11y = 22$. Therefore $y = 2$ and, hence, $x = 2 \cdot 2 - 5 = -1$. Thus, we have the solution $(x, y) = (-1, 2)$.

(b) Solving the first equation for x we have $2x = 3y - 1$, whence $x = \frac{3}{2}y - \frac{1}{2}$. Substituting into the second equation, we get $8(\frac{3}{2}y - \frac{1}{2}) - 12y = -4$, i.e., $12y - 4 - 12y = -4$, whence $0 = 0$. Thus, our system has infinitely many solutions, given by $(x, y) = (\frac{3}{2}y - \frac{1}{2}, y)$, y any real number. ■

Problem 4 *Solve the system* $\left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 11 \\ -x & + & 2y & + & z & = & -3 \\ 2x & - & 3y & + & 2z & = & 20 \end{array}\right\}$ *by using allowable operations on the equations (Gauss elimination).*

Solution:

$$\left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 11 \\ -x & + & 2y & + & z & = & -3 \\ 2x & - & 3y & + & 2z & = & 20 \end{array}\right\} \longrightarrow \left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 11 \\ & & & & 2z & = & 8 \\ & & y & & & = & -2 \end{array}\right\} \longrightarrow$$

$$\left\{\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 11 \\ & & & & z & = & 4 \\ & & y & & & = & -2 \end{array}\right\} \longrightarrow \left\{\begin{array}{rcrcrcr} x & & & & & = & 3 \\ & & & & z & = & 4 \\ & & y & & & = & -2 \end{array}\right\}$$

Thus, the solution is $(x, y, z) = (3, -2, 4)$. ■

Problem 5 *Solve the system* $\left\{\begin{array}{rcrcrcr} x & + & y & + & z & = & 4 \\ -x & - & 2y & + & 3z & = & 9 \\ 2x & + & y & - & 2z & = & -3 \end{array}\right\}$ *by using the Gauss-Jordan method (matrix row operations).*

Solution:

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 4 \\ -1 & -2 & 3 & 9 \\ 2 & 1 & -2 & -3 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 1 & 1 & 4 \\ 0 & -1 & 4 & 13 \\ 0 & -1 & -4 & -11 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 1 & 1 & 4 \\ 0 & 1 & -4 & -13 \\ 0 & -1 & -4 & -11 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 5 & 17 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & -8 & -24 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & 5 & 17 \\ 0 & 1 & -4 & -13 \\ 0 & 0 & 1 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right].$$

Thus, the solution is $(x, y, z) = (2, -1, 3)$. ■

Problem 6 *Solve the system* $\left\{\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 1 \\ -3x & - & y & + & z & = & -13 \\ 2x & + & 4y & - & 2z & = & 2 \end{array}\right\}$ *by using the Gauss-Jordan method (matrix row operations).*

Solution:

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ -3 & -1 & 1 & -13 \\ 2 & 4 & -2 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ 0 & 5 & -2 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 2 & -1 & 1 \\ 0 & 1 & -\frac{2}{5} & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{rrr|r} 1 & 0 & -\frac{1}{5} & 5 \\ 0 & 1 & -\frac{2}{5} & -2 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Thus, we have infinitely many solutions given by $(x, y, z) = (5 + \frac{1}{5}z, -2 + \frac{2}{5}z, z)$, with z any real number. ■

Problem 7 *Let* $A = \left[\begin{array}{rr} -1 & -2 \\ 5 & -1 \end{array}\right]$ *and* $B = \left[\begin{array}{rr} 3 & -7 \\ -8 & 2 \end{array}\right]$. *Compute* $A + B, A - B$ *and* $3A - 2B$.

Solution:
We have

$$A+B=\begin{bmatrix} -1 & -2 \\ 5 & -1 \end{bmatrix}+\begin{bmatrix} 3 & -7 \\ -8 & 2 \end{bmatrix}=\begin{bmatrix} 2 & -9 \\ -3 & 1 \end{bmatrix}.$$

$$A-B=\begin{bmatrix} -1 & -2 \\ 5 & -1 \end{bmatrix}-\begin{bmatrix} 3 & -7 \\ -8 & 2 \end{bmatrix}=\begin{bmatrix} -4 & 5 \\ 13 & -3 \end{bmatrix}.$$

$$3A-2B=3\begin{bmatrix} -1 & -2 \\ 5 & -1 \end{bmatrix}-2\begin{bmatrix} 3 & -7 \\ -8 & 2 \end{bmatrix}=\begin{bmatrix} -3 & -6 \\ 15 & -3 \end{bmatrix}-\begin{bmatrix} 6 & -14 \\ -16 & 4 \end{bmatrix}=\begin{bmatrix} -9 & 8 \\ 31 & -7 \end{bmatrix}.$$

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Problem 8 *Let* $A=\begin{bmatrix} 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}$ *and* $B=\begin{bmatrix} -1 & 2 & -7 \\ 3 & 2 & 10 \end{bmatrix}$. *Compute* $A-B$ *and* $-2A+5B$.

Solution:

$$A-B=\begin{bmatrix} 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}-\begin{bmatrix} -1 & 2 & -7 \\ 3 & 2 & 10 \end{bmatrix}=\begin{bmatrix} 2 & -2 & 10 \\ -4 & -5 & -3 \end{bmatrix}.$$

$$-2A+5B=-2\begin{bmatrix} 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}+5\begin{bmatrix} -1 & 2 & -7 \\ 3 & 2 & 10 \end{bmatrix}=$$

$$\begin{bmatrix} -2 & 0 & -6 \\ 2 & 6 & -14 \end{bmatrix}+\begin{bmatrix} -5 & 10 & -35 \\ 15 & 10 & 50 \end{bmatrix}=\begin{bmatrix} -7 & 10 & -41 \\ 17 & 16 & 36 \end{bmatrix}.$$

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