HOMEWORK 2 - MATH 341

DUE DATE: Tuesday, February 11

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Show that $G = \mathbf{C}^* = \mathbf{C} - \{0\}$ under complex multiplication forms a group.
   (b) Construct the group table of V and Q_8 and determine whether they are Abelian.
   (c) Find two elements a, b in S_3 such that $(ab)^2 \neq a^2b^2$.
2. (a) Show that if every element of a group G is equal to its inverse, then G is Abelian.
   (b) Let G be a finite Abelian group such that for all $a \in G, a \neq e$, we have $a^2 \neq e$. If $a_1, a_2, \ldots, a_n$ are all the elements of G with no repetitions, evaluate the product $a_1 a_2 \ldots a_n$.
3. (a) Show that the nonzero elements of $\mathbf{Z}_p$, where p is a prime, form a group under multiplication mod p.
   (b) (**Wilson's Theorem**) Prove that if p is a prime, then $(p-1)! \equiv -1 \bmod p$.
4. (a) Let $G = \{a + b\sqrt{2} : a, b \in \mathbf{Q}\}$. Show that G is a subgroup of $\mathbb{R}$ under addition.
   (b) Let $G = \{a + bi : a, b \in \mathbb{R}, a^2 + b^2 = 1\}$. Determine whether or not G is a subgroup of $\mathbf{C}^*$ under multiplication.
5. (a) Show that if H and K are subgroups of G, then $H \cap K$ is also a subgroup of G.
   (b) Let G be a group, $a \in G$. Show that the centralizer $C(a) = G$ if and only if $a \in Z(G)$, the center of G.