EXAM 1 - MATH 490

Friday, February 14, 2003
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Read each problem very carefully before starting to solve it. Each question is worth 8 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Give the definitions of a **one-one** and of an **onto** function.
   (b) Let $f : A \to B$ be a given one-one function and let $\{X_\alpha\}_{\alpha \in I}$ be an indexed family of subsets of A. Prove that $f(\cap_{\alpha \in I} X_\alpha) = \cap_{\alpha \in I} f(X_\alpha)$
2. (a) Give the definition of a **relation on** a set A. Also give the definition of an **equivalence relation** on A.
   (b) Let X be the set of functions from the real numbers into the real numbers possessing continuous derivatives. Let R be the subset of $X \times X$ consisting of those pairs (f, g) such that $Df = Dg$ where D maps a function into its derivative. Prove that R is an equivalence relation and describe an equivalence class $\pi(f)$.
3. (a) Give the definition of a **metric space**.
   (b) Let (X, d) be a metric space. Let k be a positive real number and set $d_k(x, y) = k \cdot d(x, y)$. Prove that (X, d_k) is a metric space.
4. (a) Define the function $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x_1, x_2) = x_1 + x_2$. Prove that f is continuous, where the distance function on $\mathbb{R}^2$ is
   $$d((x_1, x_2), (y_1, y_2)) = \max_{i=1,2}\{|x_i - y_i|\}.$$
   (b) Let (X, d) be a metric space. Define a distance function d^* on $X \times X$ by
   $$d^*(x, y) = \max_i\{d_i(x_i, y_i)\}.$$
   Prove that the function $d : (X \times X, d^*) \to (\mathbb{R}, d)$ is continuous.
5. Let (X, d_1) and (Y, d_2) be metric spaces. Let $f : X \to Y$ be continuous. Define a distance function d on $X \times Y$ in the standard manner. Prove that the graph Γ_f of f is a closed subset of $(X \times Y, d)$.