

EXAM 3 - MATH 490

Friday, April 18, 2003

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 8 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

- Give the definition of the **product** of finitely many topological spaces.
 - Prove that a subset F of $X = \prod_{i=1}^n X_i$ is closed if and only if F is an intersection of sets, each of which is a finite union of sets of the form $F_1 \times \cdots \times F_n$, where each F_i is a closed subset of X_i .
- Give the definition of a **category**.
 - Let C be an arbitrary category and X an object in C . Verify that the set of isomorphisms in $H(X, X)$ is a group under the operation of category composition.
- Give the definition of a **connected topological space**.
 - Show that the set \mathbf{Q} of the rational numbers with the subspace topology inherited from the usual topology of the real numbers is disconnected.
- Give a function f from a closed interval of real numbers with the usual topology into the set of real numbers, also with the usual topology, with a connected graph that is not continuous.
 - Let X be the set of real numbers with the topology $\mathcal{T} = \{U \subset X : 0 \in U\} \cup \{\emptyset\}$. Is the space (X, \mathcal{T}) connected? How about the subspace $X - \{0\}$?
- Give the definition of a **path-connected space**.
 - Let $\{X_\alpha\}_{\alpha \in A}$ be an indexed family of topological spaces and set $X = \prod_{\alpha \in A} X_\alpha$. For each $\alpha \in A$ let $f_\alpha : I \rightarrow X_\alpha$ be a path in X_α . Set $(f_A(t))(\alpha) = f_\alpha(t)$ so that $f_A : I \rightarrow X$. Prove that f_A is a path in X . Prove that if each X_α is path-connected, so is X .