HOMEWORK 3 - MATH 490

DUE DATE: Monday, February 24
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work.

GOOD LUCK!!

1. Let $(X, \mathcal{T})$ be a topological space that is metrizable. Prove that for each pair a, b of distinct points of X, there are open sets O_a and O_b containing a and b respectively, such that $O_a \cap O_b = \emptyset$. Prove that the topological space of Example 7 on page 72 is not metrizable.

2. Let $(X, \mathcal{T})$ be a topological space. Prove that $\emptyset, X$ are closed sets, that a finite union of closed sets is a closed set, and that an arbitrary intersection of closed sets is a closed set.

3. Prove that in a discrete topological space, each subset is simultaneously open and closed.

4. A family $\{A_\alpha\}_{\alpha \in I}$ of subsets is said to be *mutually disjoint* if for each distinct pair β, γ of indices $A_\beta \cap A_\gamma = \emptyset$. Prove that for each subset A of a topological space $(X, \mathcal{T})$, the three sets $\text{Int}(A), \text{Bdry}(A)$ and $\text{Int}(C(A))$ are mutually disjoint and that $X = \text{Int}(A) \cup \text{Bdry}(A) \cup \text{Int}(C(A))$.

5. In the real line prove that the boundary of the open interval (a, b) and the boundary of the closed interval $[a, b]$ is $\{a, b\}$.

6. Let A be a subset of a topological space. Prove that $\text{Bdry}(A) = \emptyset$ if and only if A is open and closed.

7. A subset A of a topological space $(X, \mathcal{T})$ is said to be *dense in* X if $\bar{A} = X$. Prove that if for each open set O we have $A \cap O = \emptyset$, then A is dense in X.

8. Let a function $f : X \to Y$ be given. Prove that $f : (X, 2^X) \to (Y, \mathcal{T}')$ is always continuous, as is $f : (X, \mathcal{T}) \to (Y, \{\emptyset, Y\})$, where $\mathcal{T}'$ is any topology on Y and $\mathcal{T}$ is any topology on X.