HOMEWORK 7 - MATH 151

DUE DATE: Monday, March 29

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each homework problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the domain, the intercepts (if possible), the vertical and horizontal asymptotes (if they exist), intervals of monotonicity and relative extrema, intervals of concavity and inflection points and then roughly sketch the graph of the function:
   (a) $f(x) = x^4 - 2x^2 - 12$
   (b) $f(x) = \frac{x-1}{x^2-4}$
   (c) $f(x) = \sin x + \cos x$
   (d) $f(x) = x^2 e^{-2x}$
   (e) $f(x) = \frac{\ln x}{\sqrt{x}}$
2. Let $s = \frac{100}{t^2+12}$ be the position function of a particle moving along a coordinate line, where s is in feet and t in seconds. Find the maximum speed of the particle for $t \geq 0$ and find the direction of the motion of the particle when it has its maximum speed.
3. Let $s_A = 15t^2 + 10t + 20$ and $s_B = 5t^2 + 40t, t \geq 0$, be the position functions of cars A and B that are moving along parallel straight lines of a highway.
   (a) How far is car A ahead of car B when $t = 0$?
   (b) At what instants of time are the cars next to each other?
   (c) At what instant of time do they have the same velocity? Which car is ahead at this instant?
4. Find the absolute maximum and minimum values of f on the given closed interval, and state where those values occur.
   (a) $f(x) = 8x - x^2; [0, 6]$
   (b) $f(x) = (x^2 + x)^{2/3}; [-2, 3]$
   (c) $f(x) = \sin x - \cos x; [0, \pi]$
   (d) $f(x) = x^4 + 4x; (-\infty, +\infty)$
   (e) $f(x) = \frac{x+3}{x-3}; [-5, 5]$
5. If f is periodic, then the locations of all absolute extrema on $(-\infty, +\infty)$ can be obtained by finding the locations of the absolute extrema for one period and using the periodicity to locate the rest. Do this analysis for

(a) $f(x) = 2 \sin 2x + \sin 4x$

(b) $f(x) = 3 \cos \frac{x}{3} + 2 \cos \frac{x}{2}$

6. One way of proving that $f(x) \leq g(x)$ for all x in a given interval is to show that $0 \leq g(x) - f(x)$ for all x in the interval; and one way of showing the latter inequality is to show that the absolute minimum value of $g(x) - f(x)$ on the interval is nonnegative. Do this for

   (a) $\sin x \leq x$ for all x in $[0, 2\pi]$.

   (b) $\cos x \geq 1 - \frac{x^2}{2}$ for all x in $[0, 2\pi]$.

7. A wire of length 12in can be bent into a circle, bent into a square, or cut into two pieces to make both a circle and a square. How much wire should be used for the circle if the total area enclosed by the figure(s) is to be

   (a) a maximum?

   (b) a minimum?

8. (a) Find the height and radius of the cone of slant height L whose volume is as large as possible.

   (b) Find a point on the curve $x = 2y^2$ closest to $(0, 9)$.