EXAM 3 - MATH 216
Thursday, March 18, 2004
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Read each problem very carefully before starting to solve it. Each question is worth 2 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Show that, if the probability that a coin comes up tails is p, then the expected number of flips of that coin until it comes up tails for the first time is $\frac{1}{p}$.

2. Give the definition of the variance $V(X)$ of a random variable X and then show that $V(X) = E(X^2) - E(X)^2$.

3. In a TV coin game, a biased coin is used with probability of tails $\frac{2}{3}$. If this coin comes up heads, you win \$5, whereas, if it comes up tails, you win only \$1. During the game the coin is tossed n times. Let X_n be the random variable denoting the amount a player wins, when n tosses take place. Find the expected value of X_n.

4. Solve the recurrence relations using the solution theorems.

    (a) $a_n = a_{n-1} + 2n + 3, a_0 = 4$

    (b) $a_n = 2a_{n-1} - 1, a_0 = 1$

5. (a) Find a recurrence relation for the number of bit strings of length n that contain a pair of consecutive 0's. (Please, explain!!)

    (b) What are the initial conditions?

    (c) How many bit strings of length seven contain two consecutive 0's?