HOMEWORK 1 - MATH 216

DUE DATE: Tuesday, January 27
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. (a) How many different three-letter initials can people have?
   (b) How many different three-letter initials are there that begin with A?
   (c) How many strings are there of four lowercase letters that have the letter x in them?

2. How many positive integers between 100 and 999 inclusive
   (a) are divisible by 7?
   (b) are odd?
   (c) have the same three decimal digits?
   (d) are not divisible by 4?
   (e) are divisible by 3 or 4?
   (f) are not divisible by either 3 or 4?
   (g) are divisible by 3 but not by 4?
   (h) are divisible by 3 and 4?

3. How many strings of four decimal digits
   (a) do not contain the same digit twice?
   (b) end with an even digit?
   (c) have exactly three digits that are 9's?

4. How many license plates can be made using either three letters followed by three digits or four letters followed by two digits?

5. A bowl contains ten red balls and ten blue balls. A woman selects balls at random without looking at them.
   (a) How many balls must she select to be sure of having at least three balls of the same color?
   (b) How many balls must she select to be sure of having at least three blue balls?

6. (a) Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9?
   (b) Is the conclusion in the first part true if four integers are selected rather than five?

7. How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

8. (a) Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence $22, 5, 7, 2, 23, 10, 15, 21, 3, 17$

   (b) Construct a sequence of 16 positive integers that has no increasing or decreasing subsequence of five terms.

9. A coin is flipped ten times where each flip comes up either heads or tails. How many possible outcomes

   (a) exactly three 0's?

   (b) more 0's than 1's?

   (c) at least seven 1's?

   (d) at least three 1's?

10. How many permutations of the letters ABCDEFGH contain

    (a) the string ED?

    (b) the string CDE?

    (c) the strings BA and FGH?

    (d) the strings AB,DE and GH?

    (e) the strings CAB and BED?

    (f) the strings BCA and ABF?

11. A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

12. Seven women and nine men are on the faculty in the mathematics department at a school.

    (a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?

    (b) How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be in the committee?

13. How many strings of six lower case letters from the English alphabet contain

    (a) the letter a?

    (b) the letters a and b?

    (c) the letters a and b in consecutive positions with a preceding b with all the letters distinct?

    (d) the letters a and b, where a is somewhere to the left of b in the string, with all the letters distinct?

14. Find the coefficient of

(a) x^5y^8 in $(x+y)^{13}$

(b) x^9 in $(2-x)^{19}$

15. Prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$ whenever n, r and k are nonnegative integers with $r \leq n$ and $k \leq r$, both by a combinatorial and by an algebraic argument.

16. Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$ using both a combinatorial and an algebraic argument.