HOMEWORK 4 - MATH 216
DUE DATE: After basics on graphs are covered.
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. A survey of households in the United States reveals that 96% have at least one television set, 98% have telephone service and 95% have telephone service and at least one television set. hat percentage of households in the United States have neither telephone service nor a television set?

2. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set if

   (a) the sets are pairwise disjoint.
   (b) there are 50 common elements in each pair of sets and no elements in all three sets.
   (c) there are 50 common elements in each pair of sets and 25 elements in all three sets.
   (d) the sets are equal.

3. Find the number of positive integers not exceeding 100 that are either odd or the square of an integer.

4. Find a formula for the probability of the union of five events in a sample space if no four of them can occur at the same time.

5. How many solutions does the equation $x_1 + x_2 + x_3 = 13$ have where x_1, x_2 and x_3 are nonnegative integers less than 6?

6. Find the number of primes less than 200 using the principle of inclusion-exclusion.

7. In how many ways can seven different jobs be assigned to four different employees so that each employee is assigned at least one job and the most difficult job is assigned to the best employee?

8. A machine that inserts letters into envelopes goes haywire and inserts letters randomly into envelopes. What is the probability that in a group of 100 letters

   (a) no letter is put into the correct envelope?
   (b) exactly one letter is put into the correct envelope?
   (c) exactly 98 letters are put into the correct envelopes?
   (d) exactly 99 letters are put into the correct envelopes?
   (e) all letters are put into the correct envelopes?

9. The intersection graph of a collection of sets $A_1, A_2, \ldots, A_n$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets:

   (a) $A_1 = \{\ldots, -4, -3, -2, -1, 0\}, A_2 = \{\ldots, -2, -1, 0, 1, 2, \ldots\},$
   $A_3 = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}, A_4 = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\},$
   $A_5 = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$

   (b) $A_1 = \{x : x < 0\}, A_2 = \{x : -1 < x < 0\}, A_3 = \{x : 0 < x < 1\},$
   $A_4 = \{x : -1 < x < 1\}, A_5 = \{x : x > -1\}, A_6 = \mathbb{R}.$

10. We can use a graph to represent whether two people were alive at the same time. Draw such a graph to represent whether each pair of the mathematicians and computer scientists with biographies in the first four chapters of your book who died before 1900 were contemporaneous.

11. Construct an influence graph for the board members of a company if the president can influence the director of research and development, the director of marketing and the director of operations; the director of research and development can influence the director of operations; the director of marketing can influence the director of operations; and no one can influence or be influenced by the chief financial officer.

12. Can a simple graph exist with 15 vertices each of degree 5?

13. What does the degree of a vertex represent in a collaboration graph? What do isolated vertices and pendant vertices represent? Answer the same questions for the Hollywood graph.

14. How many subgraphs with at least one vertex does the following have? (a) K_2 (b) K_3 (c) W_3

15. (a) For which values of n are K_n, C_n, W_n, Q_n regular?
   (b) For which values of m, n is $K_{m,n}$ regular?
   (c) How many vertices does a regular graph of degree 4 with 10 edges have?

16. Find the union of the given pair of simple graphs

   (a)

   (b)